Proof $(\mathsf{RLD})_{\mathrm{in}}(f\cap(\mathcal{A}\times^{\mathsf{FCD}}\mathcal{B})) = (\mathsf{RLD})_{\mathrm{in}}f\cap(\mathsf{RLD})_{\mathrm{in}}(\mathcal{A}\times^{\mathsf{FCD}}\mathcal{B}) = ((\mathsf{RLD})_{\mathrm{in}}f)\cap(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B})$. □

Corollary 18 $(\mathsf{RLD})_{\mathrm{in}}(f|_{\mathcal{A}}) = ((\mathsf{RLD})_{\mathrm{in}}f)|_{\mathcal{A}}$ *for every funcoid* f *and f.o.* $\mathcal{A}$.

Conjecture 19 $(\mathsf{RLD})_{\mathrm{in}}$ *is not a lower adjoint (in general).*

Conjecture 20 $(\mathsf{RLD})_{\mathrm{out}}$ *is neither a lower adjoint nor an upper adjoint (in general).*

See also the corollary 23 below.

6 Continuous morphisms

This section uses the apparatus from the section "Partially ordered dagger categories".

6.1 Traditional definitions of continuity

In this section we will show that having a funcoid or reloid $\uparrow f$ corresponding to a function f we can express continuity of it by the formula $\uparrow f\circ\mu\subseteq\nu\circ\uparrow f$ (or similar formulas) where μ and ν are some spaces.

6.1.1 Pre-topology

Let μ and ν are funcoids representing some pre-topologies. By definition a function f is continuous map from μ to ν in point a iff

$$\forall\epsilon\in\operatorname{up}\langle\nu\rangle^*\{fa\}\exists\delta\in\operatorname{up}\langle\mu\rangle^*\{a\}:\langle f\rangle\delta\subseteq\epsilon.$$

Equivalently transforming this formula we get:

$$\begin{gathered}\forall\epsilon\in\operatorname{up}\langle\nu\rangle^*\{fa\}:\left\langle\uparrow^{\mathsf{FCD}(\operatorname{Src}\mu;\operatorname{Dst}\nu)}f\right\rangle\langle\mu\rangle\uparrow^{\operatorname{Src}\mu}\{a\}\subseteq\epsilon;\\ \left\langle\uparrow^{\mathsf{FCD}(\operatorname{Src}\mu;\operatorname{Dst}\nu)}f\right\rangle\langle\mu\rangle\uparrow^{\operatorname{Src}\mu}\{a\}\subseteq\langle\nu\rangle^*\{fa\};\\ \left\langle\uparrow^{\mathsf{FCD}(\operatorname{Src}\mu;\operatorname{Dst}\nu)}f\right\rangle\langle\mu\rangle\uparrow^{\operatorname{Src}\mu}\{a\}\subseteq\langle\nu\rangle\left\langle\uparrow^{\mathsf{FCD}(\operatorname{Src}\mu;\operatorname{Dst}\nu)}f\right\rangle\uparrow^{\operatorname{Src}\mu}\{a\};\\ \left\langle\uparrow^{\mathsf{FCD}(\operatorname{Src}\mu;\operatorname{Dst}\nu)}f\circ\mu\right\rangle\uparrow^{\operatorname{Src}\mu}\{a\}\subseteq\left\langle\nu\circ\uparrow^{\mathsf{FCD}(\operatorname{Src}\mu;\operatorname{Dst}\nu)}f\right\rangle\uparrow^{\operatorname{Src}\mu}\{a\}.\end{gathered}$$

So f is a continuous map from μ to ν in every point of its domain iff

$$\uparrow^{\mathsf{FCD}(\operatorname{Src}\mu;\operatorname{Dst}\nu)}f\circ\mu\subseteq\nu\circ\uparrow^{\mathsf{FCD}(\operatorname{Src}\mu;\operatorname{Dst}\nu)}f.$$