from this by the lemma 4 (taking in account that $\{G \circ F \mid F \in \operatorname{up} f, G \in \operatorname{up} g\}$ and $\operatorname{up} \bigcap \left\{ \uparrow^{\mathsf{RLD}(\operatorname{Src} f; \operatorname{Dst} g)} (G \circ F) \mid F \in \operatorname{up} f, G \in \operatorname{up} g \right\}$ are filter bases)

$$\bigcap \left\{ \uparrow^{\operatorname{Dst} g} \langle H \rangle X \mid H \in \operatorname{up} \bigcap \left\{ \uparrow^{\mathsf{RLD}(\operatorname{Src} f; \operatorname{Dst} g)} (G \circ F) \mid F \in \operatorname{up} f, G \in \operatorname{up} g \right\} \right\} = \bigcap \left\{ \uparrow^{\operatorname{Dst} g} \langle G \circ F \rangle X \mid F \in \operatorname{up} f, G \in \operatorname{up} g \right\}.$$

On the other side

$$\begin{aligned} \langle ((\mathsf{FCD})g) \circ ((\mathsf{FCD})f) \rangle^* X &= \langle (\mathsf{FCD})g \rangle \langle (\mathsf{FCD})f \rangle^* X \\ &= \langle (\mathsf{FCD})g \rangle \bigcap \left\{ \uparrow^{\operatorname{Dst} f} \langle F \rangle X \mid F \in \operatorname{up} f \right\} \\ &= \bigcap \left\{ \left\langle \uparrow^{\mathsf{FCD}(\operatorname{Src} g; \operatorname{Dst} g)} G \right\rangle \bigcap \left\{ \uparrow^{\operatorname{Dst} f} \langle F \rangle X \mid F \in \operatorname{up} f \right\} \mid G \in \operatorname{up} g \right\}. \end{aligned}$$

Let's prove that $\{\langle F \rangle X \mid F \in \operatorname{up} f\}$ is a filter base. If $A, B \in \{\langle F \rangle X \mid F \in \operatorname{up} f\}$ then $A = \langle F_1 \rangle X$ and $B = \langle F_2 \rangle X$ where $F_1, F_2 \in \operatorname{up} f$. $A \cap B \supseteq \langle F_1 \cap F_2 \rangle X \in \{\langle F \rangle X \mid F \in \operatorname{up} f\}$. So $\{\langle F \rangle X \mid F \in \operatorname{up} f\}$ is really a filter base.

By the theorem 8 we have

$$\left\langle \uparrow^{\mathsf{FCD}(\operatorname{Src} g; \operatorname{Dst} g)} G \right\rangle \bigcap \left\{ \uparrow^{\operatorname{Dst} f} \langle F \rangle X \mid F \in \operatorname{up} f \right\} = \bigcap \left\{ \uparrow^{\operatorname{Dst} g} \langle G \rangle \langle F \rangle X \mid F \in \operatorname{up} f \right\}.$$

So continuing the above equalities,

$$\begin{aligned} \langle ((\mathsf{FCD})g) \circ ((\mathsf{FCD})f) \rangle^* X &= \bigcap \left\{ \bigcap \left\{ \uparrow^{\operatorname{Dst} g} \langle G \rangle \langle F \rangle X \mid F \in \operatorname{up} f \right\} \mid G \in \operatorname{up} g \right\} \\ &= \bigcap \left\{ \uparrow^{\operatorname{Dst} g} \langle G \rangle \langle F \rangle X \mid F \in \operatorname{up} f, G \in \operatorname{up} g \right\} \\ &= \bigcap \left\{ \uparrow^{\operatorname{Dst} g} \langle G \circ F \rangle X \mid F \in \operatorname{up} f, G \in \operatorname{up} g \right\}. \end{aligned}$$

Combining these equalities we get $\langle (\mathsf{FCD})(g \circ f) \rangle^* X = \langle ((\mathsf{FCD})g) \circ ((\mathsf{FCD})f) \rangle^* X$ for every set X. □

Corollary 15

1. $(\mathsf{FCD})\, f$ *is a monovalued funcoid if* f *is a monovalued reloid.*

2. $(\mathsf{FCD})\, f$ *is an injective funcoid if* f *is an injective reloid.*

Proof We will prove only the first as the second is dual. Let f is a monovalued reloid. Then $f \circ f^{-1} \subseteq I^{\mathsf{RLD}(\operatorname{Dst} f)}$; $(\mathsf{FCD}) \left(f \circ f^{-1}\right) \subseteq I^{\mathsf{FCD}(\operatorname{Dst} f)}$; $(\mathsf{FCD})\, f \circ ((\mathsf{FCD})\, f)^{-1} \subseteq I^{\mathsf{FCD}(\operatorname{Dst} f)}$ that is $(\mathsf{FCD})\, f$ is a monovalued funcoid. □

Proposition 32 $(\mathsf{FCD}) I^{\mathsf{RLD}}_{\mathcal{A}} = I^{\mathsf{FCD}}_{\mathcal{A}}$ *for every f.o.* $\mathcal{A}$.