

* theorem 40 in [15].

Thus $\text{Compl } f$ is principal. \square

Theorem 60 $\text{Compl } \text{CoCompl } f = \text{CoCompl } \text{Compl } f = \text{Cor } f$ for every reloid f .

Proof We will prove only $\text{Compl } \text{CoCompl } f = \text{Cor } f$. The rest follows from symmetry.

From the lemma $\text{Compl } \text{CoCompl } f$ is principal. It is obvious $\text{Compl } \text{CoCompl } f \subseteq f$. So to finish the proof we need to show only that for every principal reloid $F \subseteq f$ we have $F \subseteq \text{Compl } \text{CoCompl } f$.

Really, obviously $F \subseteq \text{CoCompl } f$ and thus $F = \text{Compl } F \subseteq \text{Compl } \text{CoCompl } f$. \square

Question 28. Is $\text{Compl } \text{RLD}(A; B)$ a distributive lattice? Is $\text{Compl } \text{RLD}(A; B)$ a co-brouwerian lattice?

Conjecture 12 Let A, B, C are small sets. If $f \in \text{RLD}(B; C)$ is a complete reloid and $R \in \mathcal{P}\text{RLD}(A; B)$ then

$$f \circ \bigcup R = \bigcup \langle f \circ \rangle R.$$

This conjecture can be weakened:

Conjecture 13 Let A, B, C are small sets. If $f \in \text{RLD}(B; C)$ is a principal reloid and $R \in \mathcal{P}\text{RLD}(A; B)$ then

$$f \circ \bigcup R = \bigcup \langle f \circ \rangle R.$$

Conjecture 14 $\text{Compl } f = f \setminus * (\Omega^{\text{Src } f} \times_{\text{RLD}} 1^{\mathfrak{F}(\text{Dst } f)})$ for every reloid f .

5 Relationships between funcoids and reloids

5.1 Funcoid induced by a reloid

Every reloid f induces a funcoid $(\text{FCD})f \in \text{FCD}(\text{Src } f; \text{Dst } f)$ by the following formulas (for every $\mathcal{X} \in \mathfrak{F}(\text{Src } f)$, $\mathcal{Y} \in \mathfrak{F}(\text{Dst } f)$):

$$\begin{aligned} \mathcal{X} [(\text{FCD})f] \mathcal{Y} &\Leftrightarrow \forall F \in \text{up } f : \mathcal{X} [\uparrow^{\text{FCD}(\text{Src } f; \text{Dst } f)} F] \mathcal{Y}; \\ \langle (\text{FCD})f \rangle \mathcal{X} &= \bigcap \{ \langle \uparrow^{\text{FCD}(\text{Src } f; \text{Dst } f)} F \rangle \mathcal{X} \mid F \in \text{up } f \}. \end{aligned}$$

We should prove that $(\text{FCD})f$ is really a funcoid.

Proof We need to prove that

$$\mathcal{X} [(\text{FCD})f] \mathcal{Y} \Leftrightarrow \mathcal{Y} \cap \langle (\text{FCD})f \rangle \mathcal{X} \neq 0^{\mathfrak{F}(\text{Dst } f)} \Leftrightarrow \mathcal{X} \cap \langle (\text{FCD})f^{-1} \rangle \mathcal{Y} \neq 0^{\mathfrak{F}(\text{Dst } f)}.$$