Proof Let's denote R the right part of the equality to be proven.

That R is a complete reloid follows from the equality

$$f|_{\uparrow^{\operatorname{Src} f}\{\alpha\}} = \uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{RLD}} \operatorname{im}\left(f|_{\uparrow^{\operatorname{Src} f}\{\alpha\}}\right).$$

The only thing left to prove is that $g \subseteq R$ for every complete reloid g such that $g \subseteq f$.

Really let g is a complete reloid such that $g \subseteq f$. Then

$$g = \bigcup \left\{ \uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{RLD}} G(\alpha) \quad | \quad \alpha \in \operatorname{Src} f \right\}$$

for some function $G : \operatorname{Src} f \to \mathfrak{F}(\operatorname{Dst} f)$.

We have $\uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{RLD}} G(\alpha) = g|_{\uparrow^{\operatorname{Src} f}\{\alpha\}} \subseteq f|_{\uparrow^{\operatorname{Src} f}\{\alpha\}}$. Thus $g \subseteq R$. □

Conjecture 11 $\operatorname{Compl} f \cap \operatorname{Compl} g = \operatorname{Compl}(f \cap g)$ for every reloids f and g.

Theorem 59 $\operatorname{Compl}(\bigcup R) = \bigcup \langle \operatorname{Compl} \rangle R$ *for every set* $R \in \mathscr{P}\mathsf{RLD}(A; B)$ *for every small sets* A, B.

Proof

$$\begin{aligned}
\operatorname{Compl}\left(\bigcup R\right) &= \\
\bigcup \left\{ \left(\bigcup R\right)|_{\uparrow^{A}\{\alpha\}} \quad | \quad \alpha \in A \right\} &= \quad \text{(theorem 40 in [15])} \\
\bigcup \left\{ \bigcup \left\{ f|_{\uparrow^{A}\{\alpha\}} \quad | \quad \alpha \in A \right\} \quad | \quad f \in R \right\} &= \\
\bigcup \langle \operatorname{Compl} \rangle R.
\end{aligned}$$

□

Lemma 3 *Completion of a co-complete reloid is principal.*

Proof Let f is a co-complete reloid. Then there is a function $F : \operatorname{Dst} f \to \mathfrak{F}(\operatorname{Src} f)$ such that

$$f = \bigcup \left\{ F(\alpha) \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f} \{\alpha\} \quad | \quad \alpha \in \operatorname{Dst} f \right\}.$$

So

$$\begin{aligned}
\operatorname{Compl} f &= \\
\bigcup \left\{ \left(\bigcup \left\{ F(\alpha) \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f} \{\alpha\} \quad | \quad \alpha \in \operatorname{Dst} f \right\} \right)|_{\uparrow^{\operatorname{Src} f}\{\beta\}} \quad | \quad \beta \in \operatorname{Src} f \right\} &= \\
\bigcup \left\{ \left(\bigcup \left\{ F(\alpha) \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f} \{\alpha\} \quad | \quad \alpha \in \operatorname{Dst} f \right\} \right) \cap \left(\uparrow^{\operatorname{Src} f} \{\beta\} \times^{\mathsf{RLD}} 1^{\mathfrak{F}(\operatorname{Dst} f)} \right) \quad | \quad \beta \in \operatorname{Src} f \right\} &= \quad (*) \\
\bigcup \left\{ \bigcup \left\{ \left(F(\alpha) \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f} \{\alpha\} \right) \cap \left(\uparrow^{\operatorname{Src} f} \{\beta\} \times^{\mathsf{RLD}} 1^{\mathfrak{F}(\operatorname{Dst} f)} \right) \quad | \quad \alpha \in \operatorname{Dst} f \right\} \quad | \quad \beta \in \operatorname{Src} f \right\} &= \\
\bigcup \left\{ \bigcup \left\{ \uparrow^{\operatorname{Src} f} \{\beta\} \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f} \{\alpha\} \quad | \quad \alpha \in \operatorname{Dst} f \right\} \quad | \quad \beta \in \operatorname{Src} f, \uparrow^{\operatorname{Src} f} \{\beta\} \subseteq F(\alpha) \right\}.
\end{aligned}$$