Conjecture 10 *Composition of complete reloids is complete.*

Theorem 57

1. *For a complete reloid f there exists exactly one function $F \in \mathfrak{F}(\operatorname{Dst} f)^{\operatorname{Src} f}$ such that*
$$f = \bigcup \left\{ \uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{RLD}} F(\alpha) \quad | \quad \alpha \in \operatorname{Src} f \right\}.$$

2. *For a co-complete reloid f there exists exactly one function $F \in \mathfrak{F}(\operatorname{Src} f)^{\operatorname{Dst} f}$ such that*
$$f = \bigcup \left\{ F(\alpha) \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f} \{\alpha\} \quad | \quad \alpha \in \operatorname{Dst} f \right\}.$$

Proof We will prove only the first as the second is similar. Let

$$f = \bigcup \left\{ \uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{RLD}} F(\alpha) \quad | \quad \alpha \in \operatorname{Src} f \right\} = \bigcup \left\{ \uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{RLD}} G(\alpha) \quad | \quad \alpha \in \operatorname{Src} f \right\}$$

for some $F, G \in \mathfrak{F}(\operatorname{Dst} f)^{\operatorname{Src} f}$. We need to prove $F = G$. Let $\beta \in \operatorname{Src} f$.

$$f \cap \left(\uparrow^{\operatorname{Src} f} \{\beta\} \times^{\mathsf{RLD}} 1^{\mathfrak{F}(\operatorname{Dst} f)} \right) = \text{(theorem 40 in [15])}$$
$$\bigcup^{\mathsf{RLD}} \left\{ \left(\uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{RLD}} F(\alpha) \right) \cap^{\mathsf{RLD}} \left(\uparrow^{\operatorname{Src} f} \{\beta\} \times 1^{\mathfrak{F}(\operatorname{Dst} f)} \right) \quad | \quad \alpha \in \operatorname{Src} f \right\} =$$
$$\uparrow^{\operatorname{Src} f} \{\beta\} \times^{\mathsf{RLD}} F(\beta).$$

Similarly $f \cap \left(\uparrow^{\operatorname{Src} f} \{\beta\} \times 1^{\mathfrak{F}(\operatorname{Dst} f)} \right) = \uparrow^{\operatorname{Src} f} \{\beta\} \times^{\mathsf{RLD}} G(\beta)$. Thus $\uparrow^{\operatorname{Src} f} \{\beta\} \times^{\mathsf{RLD}} F(\beta) = \uparrow^{\operatorname{Src} f} \{\beta\} \times^{\mathsf{RLD}} G(\beta)$ and so $F(\beta) = G(\beta)$. □

Definition 54 ***Completion*** *and* ***co-completion*** *of a reloid $f \in \mathsf{RLD}(A; B)$ are defined by the formulas:*

$$\operatorname{Compl} f = \operatorname{Cor}^{(\mathsf{RLD}(A;B); \operatorname{Compl} \mathsf{RLD}(A;B))} f \quad \text{and} \quad \operatorname{CoCompl} f = \operatorname{Cor}^{(\mathsf{RLD}(A;B); \operatorname{CoCompl} \mathsf{RLD}(A;B))} f.$$

Theorem 58 *Atoms of the lattice* $\operatorname{Compl} \mathsf{RLD}(A; B)$ *are exactly direct products of the form $\uparrow^A \{\alpha\} \times^{\mathsf{RLD}} b$ where $\alpha \in A$ and b is an atomic f.o. on B.*

Proof First, it's easy to see that $\uparrow^A \{\alpha\} \times^{\mathsf{FCD}} b$ are elements of $\operatorname{Compl} \mathsf{RLD}(A; B)$. Also $0^{\mathsf{RLD}(A;B)}$ is an element of $\operatorname{Compl} \mathsf{RLD}$.

$\uparrow^A \{\alpha\} \times^{\mathsf{RLD}} b$ are atoms of $\operatorname{Compl} \mathsf{FCD}$ because these are atoms of RLD.

It remains to prove that if f is an atom of $\operatorname{Compl} \mathsf{RLD}(A; B)$ then $f = \uparrow^A \{\alpha\} \times^{\mathsf{RLD}} b$ for some $\alpha \in A$ and an atomic f.o. b on B.

Suppose f is a non-empty complete reloid. Then $\uparrow^A \{\alpha\} \times^{\mathsf{RLD}} b \subseteq f$ for some $\alpha \in A$ and atomic f.o. b on B. If f is an atom then $f = \uparrow^A \{\alpha\} \times^{\mathsf{FCD}} b$. □

Obvious 27. $\operatorname{Compl} \mathsf{RLD}(A; B)$ is an atomistic lattice.

Proposition 30 $\operatorname{Compl} f = \bigcup \left\{ f|_{\uparrow^{\operatorname{Src} f} \{\alpha\}} \quad | \quad \alpha \in \operatorname{Src} f \right\}$ *for every reloid f.*