$\Rightarrow$ Let f is complete. Then take

$$G(\alpha)=\bigcup\left\{b\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Dst}f)}\quad|\quad\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}b\subseteq f\right\}$$

and we have (12) obviously.

$\Leftarrow$ Let (12) holds. Then $G(\alpha)=\bigcup\operatorname{atoms}G(\alpha)$ and thus

$$f=\bigcup\left\{\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}b\quad|\quad\alpha\in\operatorname{Src}f,b\in\operatorname{atoms}G(\alpha)\right\}$$

and so f is complete.

□

Obvious 24. Complete and co-complete reloids are convex.

Obvious 25. Principal reloids are complete and co-complete.

Obvious 26. Join (on the lattice of reloids) of complete reloids is complete.

Corollary 14 $\operatorname{Compl}\mathsf{RLD}$ *(with the induced order) is a complete lattice.*

Theorem 56 *A reloid which is both complete and co-complete is principal.*

Proof Let f is a complete and co-complete reloid. We have

$$f=\bigcup\left\{\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}G(\alpha)\quad|\quad\alpha\in\operatorname{Src}f\right\}\quad\text{and}\quad f=\bigcup\left\{H(\beta)\times^{\mathsf{RLD}}\uparrow^{\operatorname{Dst}f}\{\beta\}\quad|\quad\beta\in\operatorname{Dst}f\right\}$$

for some functions $G:\operatorname{Src}f\to\mathfrak{F}(\operatorname{Dst}f)$, $H:\operatorname{Dst}f\to\mathfrak{F}(\operatorname{Src}f)$. For every $\alpha\in\operatorname{Src}f$ we have

$$\begin{aligned}
G(\alpha) &= \\
\operatorname{im}f|_{\uparrow^{\operatorname{Src}f}\{\alpha\}} &= \\
\operatorname{im}\left(f\cap\left(\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}1^{\mathfrak{F}(\operatorname{Dst}f)}\right)\right) &= \quad (*)\\
\operatorname{im}\bigcup\left\{\left(H(\beta)\times^{\mathsf{RLD}}\uparrow^{\operatorname{Dst}f}\{\beta\}\right)\cap\left(\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}1^{\mathfrak{F}(\operatorname{Dst}f)}\right)\quad|\quad\beta\in\operatorname{Dst}f\right\} &= \\
\operatorname{im}\bigcup\left\{\left(H(\beta)\cap\uparrow^{\operatorname{Src}f}\{\alpha\}\right)\times^{\mathsf{RLD}}\uparrow^{\operatorname{Dst}f}\{\beta\}\quad|\quad\beta\in\operatorname{Dst}f\right\} &= \\
\operatorname{im}\bigcup\left\{\left(\begin{cases}\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}\uparrow^{\operatorname{Dst}f}\{\beta\} & \text{if }H(\beta)\not\asymp\uparrow^{\operatorname{Src}f}\{\alpha\}\\ 0^{\mathsf{RLD}(\operatorname{Src}f;\operatorname{Dst}f)} & \text{if }H(\beta)\asymp\uparrow^{\operatorname{Src}f}\{\alpha\}\end{cases}\right)\quad|\quad\beta\in\operatorname{Dst}f\right\} &= \\
\operatorname{im}\bigcup\left\{\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}\uparrow^{\operatorname{Dst}f}\{\beta\}\quad|\quad\beta\in\operatorname{Dst}f,H(\beta)\not\asymp\uparrow^{\operatorname{Src}f}\{\alpha\}\right\} &= \\
\operatorname{im}\bigcup\left\{\uparrow^{\mathsf{RLD}(\operatorname{Src}f;\operatorname{Dst}f)}\{(\alpha;\beta)\}\quad|\quad\beta\in\operatorname{Dst}f,H(\beta)\not\asymp\uparrow^{\operatorname{Src}f}\{\alpha\}\right\} &= \\
\bigcup\left\{\uparrow^{\operatorname{Dst}f}\{\beta\}\quad|\quad\beta\in\operatorname{Dst}f,H(\beta)\not\asymp\uparrow^{\operatorname{Src}f}\{\alpha\}\right\}.
\end{aligned}$$

* the theorem 40 from [15] was used.

Thus $G(\alpha)$ is a principal f.o. that is $G(\alpha)=\uparrow^{\operatorname{Dst}f}g(\alpha)$ for some $g:\operatorname{Src}f\to\operatorname{Dst}f$; $\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}G(\alpha)=\uparrow^{\mathsf{RLD}(\operatorname{Src}f;\operatorname{Dst}f)}(\{\alpha\}\times g(\alpha))$; f is principal as a join of principal reloids. □