1. Let f is a monovalued reloid. Then $f \circ f^{-1} \subseteq I^{\mathsf{RLD}(\operatorname{Dst} f)}$. So exists

$$h \in \operatorname{up}(f \circ f^{-1}) = \operatorname{up} \bigcap \left\{ \uparrow^{\mathsf{RLD}(\operatorname{Dst} f;\operatorname{Dst} f)} \left(F \circ F^{-1} \right) \quad | \quad F \in \operatorname{up} f \right\}$$

such that $h \subseteq I^{\mathsf{RLD}(\operatorname{Dst} f)}$. It's simple to show that $\left\{ F \circ F^{-1} \quad | \quad F \in \operatorname{up} f \right\}$ is a filter base. Consequently it exists $F \in \operatorname{up} f$ such that $F \circ F^{-1} \subseteq I_{\operatorname{Dst} f}$ that is F is a function.

2. Similar.

3. Let f is a both monovalued and injective reloid. Then by proved above there exist $F, G \in \operatorname{up} f$ such that F is monovalued and G is injective. Thus $F \cap G \in \operatorname{up} f$ is both monovalued and injective.

□

Conjecture 9 *A reloid f is monovalued iff*

$$\forall g \in \mathsf{RLD}(\operatorname{Src} f; \operatorname{Dst} f) : (g \subseteq f \Rightarrow \exists \mathcal{A} \in \mathfrak{F}(\operatorname{Src} f) : g = f|_{\mathcal{A}}).$$

4.6 Complete reloids and completion of reloids

Definition 52 *A **complete** reloid is a reloid representable as join of direct products $\uparrow^{A} \{\alpha\} \times^{\mathsf{RLD}} b$ where $\alpha \in A$ and b is an atomic f.o. on B for some small sets A and B.*

Definition 53 *A **co-complete** reloid is a reloid representable as join of direct products $a \times^{\mathsf{RLD}} \uparrow^{B} \{\beta\}$ where $\beta \in B$ and a is an atomic f.o. on A for some small sets A and B.*

I will denote the sets of complete and co-complete reloids correspondingly as $\operatorname{Compl} \mathsf{RLD}$ and $\operatorname{CoCompl} \mathsf{RLD}$.

Obvious 23. Complete and co-complete are dual.

Theorem 55

1. *A reloid $f \in \mathsf{RLD}(A; B)$ is complete iff there exists a function $G : A \to \mathfrak{F}(B)$ such that*
$$f = \bigcup \left\{ \uparrow^{A} \{\alpha\} \times^{\mathsf{RLD}} G(\alpha) \quad | \quad \alpha \in A \right\}. \tag{12}$$

2. *A reloid $f \in \mathsf{RLD}(A; B)$ is co-complete iff there exists a function $G : B \to \mathfrak{F}(A)$ such that*
$$f = \bigcup \left\{ G(\alpha) \times^{\mathsf{RLD}} \uparrow^{B} \{\alpha\} \quad | \quad \alpha \in B \right\}.$$

Proof We will prove only the first as the second is symmetric.