□

Definition 51 *I call* ***restricted identity reloid*** *for a filter object $\mathcal{A}$ the reloid*

$$I^{\mathsf{RLD}}_{\mathcal{A}} \stackrel{\mathrm{def}}{=} \left(I^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A}))}\right)|_{\mathcal{A}}.$$

Theorem 49 $I^{\mathsf{RLD}}_{\mathcal{A}} = \bigcap\left\{\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))} I_A \mid A \in \mathrm{up}\,\mathcal{A}\right\}$ *where I_A is the identity relation on a set A.*

Proof Let $K \in \mathrm{up}\bigcap\left\{\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))} I_A \mid A \in \mathrm{up}\,\mathcal{A}\right\}$, then exists $A \in \mathrm{up}\,\mathcal{A}$ such that $K \supseteq I_A$. Then

$I^{\mathsf{RLD}}_{\mathcal{A}} \subseteq\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))} \left(I_{\mathrm{Base}(\mathcal{A})}\right)\cap\left(\mathcal{A}\times^{\mathsf{RLD}} 1^{\mathfrak{F}(\mathrm{Base}(\mathcal{A}))}\right) \subseteq\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))} \left(I_{\mathrm{Base}(\mathcal{A})}\right)\cap\left(\uparrow^{\mathrm{Base}(\mathcal{A})} A\times^{\mathsf{RLD}} 1^{\mathfrak{F}(\mathrm{Base}(\mathcal{A}))}\right) =\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))} \left(I_{\mathrm{Base}(\mathcal{A})}\right) \cap \uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))} (A\times\mathrm{Base}\,(\mathcal{A})) =\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))} \left(I_{\mathrm{Base}(\mathcal{A})}\cap(A\times\mathrm{Base}\,(\mathcal{A}))\right) =\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))} I_A \subseteq\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))} K$,

Thus $K \in \mathrm{up}\, I^{\mathsf{RLD}}_{\mathcal{A}}$.

Reversely let $K \in \mathrm{up}\, I^{\mathsf{RLD}}_{\mathcal{A}} = \mathrm{up}\left(I^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A}))}\cap\left(\mathcal{A}\times^{\mathsf{RLD}} 1^{\mathfrak{F}(\mathrm{Base}(\mathcal{A}))}\right)\right)$, then exists $A \in \mathrm{up}\,\mathcal{A}$ such that $K \in \mathrm{up}\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))}\left(I_{\mathrm{Base}(\mathcal{A})}\cap(A\times\mathrm{Base}(\mathcal{A}))\right) = \mathrm{up}\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))} I_A \subseteq \mathrm{up}\bigcap\left\{\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{A}))} I_A \mid A \in \mathrm{up}\,\mathcal{A}\right\}$. □

Proposition 29 $\left(I^{\mathsf{RLD}}_{\mathcal{A}}\right)^{-1} = I^{\mathsf{RLD}}_{\mathcal{A}}$.

Proof It follows from the previous theorem. □

Theorem 50 $f|_{\mathcal{A}} = f\circ I^{\mathsf{RLD}}_{\mathcal{A}}$ *for every reloid f and $\mathcal{A}\in\mathfrak{F}(\mathrm{Src}\, f)$.*

Proof We need to prove that $f\cap\left(\mathcal{A}\times^{\mathsf{RLD}} 1^{\mathfrak{F}(\mathrm{Dst}\, f)}\right) = f\circ\bigcap\left\{\uparrow^{\mathsf{RLD}(\mathrm{Src}\, f;\mathrm{Src}\, f)} I_A \mid A \in \mathrm{up}\,\mathcal{A}\right\}$.

We have $f\circ\bigcap\left\{\uparrow^{\mathsf{RLD}(\mathrm{Src}\, f;\mathrm{Src}\, f)} I_A \mid A \in \mathrm{up}\,\mathcal{A}\right\} = \bigcap\left\{\uparrow^{\mathsf{RLD}(\mathrm{Src}\, f;\mathrm{Dst}\, f)} (F\circ I_A) \mid F\in\mathrm{up}\, f, A \in \mathrm{up}\,\mathcal{A}\right\} = \bigcap\left\{\uparrow^{\mathsf{RLD}(\mathrm{Src}\, f;\mathrm{Dst}\, f)} (F|_A) \mid F\in\mathrm{up}\, f, A \in \mathrm{up}\,\mathcal{A}\right\} =$
$\bigcap\left\{\uparrow^{\mathsf{RLD}(\mathrm{Src}\, f;\mathrm{Dst}\, f)} (F\cap(A\times\mathrm{Dst}\, f)) \mid F\in\mathrm{up}\, f, A \in \mathrm{up}\,\mathcal{A}\right\} = \bigcap\left\{\uparrow^{\mathsf{RLD}(\mathrm{Src}\, f;\mathrm{Dst}\, f)} F \mid F\in\mathrm{up}\, f\right\}\cap \bigcap\left\{\uparrow^{\mathsf{RLD}(\mathrm{Src}\, f;\mathrm{Dst}\, f)} (A\times\mathrm{Dst}\, f) \mid A \in \mathrm{up}\,\mathcal{A}\right\} = f\cap\left(\mathcal{A}\times^{\mathsf{RLD}} 1^{\mathfrak{F}(\mathrm{Dst}\, f)}\right)$. □

Theorem 51 $(g\circ f)|_{\mathcal{A}} = g\circ(f|_{\mathcal{A}})$ *for every composable reloids f and g and $\mathcal{A}\in\mathfrak{F}(\mathrm{Src}\, f)$.*

Proof $(g\circ f)|_{\mathcal{A}} = (g\circ f)\circ I^{\mathsf{RLD}}_{\mathcal{A}} = g\circ\left(f\circ I^{\mathsf{RLD}}_{\mathcal{A}}\right) = g\circ(f|_{\mathcal{A}})$. □

Theorem 52 $f\cap(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B}) = I^{\mathsf{RLD}}_{\mathcal{B}}\circ f\circ I^{\mathsf{RLD}}_{\mathcal{A}}$ *for every reloid f and $\mathcal{A}\in\mathfrak{F}(\mathrm{Src}\, f)$, $\mathcal{B}\in\mathfrak{F}(\mathrm{Dst}\, f)$.*

Proof $f\cap(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B}) = f\cap\left(\mathcal{A}\times^{\mathsf{RLD}} 1^{\mathfrak{F}(\mathrm{Dst}\, f)}\right)\cap\left(1^{\mathfrak{F}(\mathrm{Src}\, f)}\times^{\mathsf{RLD}}\mathcal{B}\right) = f|_{\mathcal{A}}\cap\left(1^{\mathfrak{F}(\mathrm{Src}\, f)}\times^{\mathsf{RLD}}\mathcal{B}\right) = \left(f\circ I^{\mathsf{RLD}}_{\mathcal{A}}\right)\cap\left(1^{\mathfrak{F}(\mathrm{Src}\, f)}\times^{\mathsf{RLD}}\mathcal{B}\right) = \left(\left(f\circ I^{\mathsf{RLD}}_{\mathcal{A}}\right)^{-1}\cap\left(1^{\mathfrak{F}(\mathrm{Src}\, f)}\times^{\mathsf{RLD}}\mathcal{B}\right)^{-1}\right)^{-1} = \left(\left(I^{\mathsf{RLD}}_{\mathcal{A}}\circ f^{-1}\right)\cap\left(\mathcal{B}\times^{\mathsf{RLD}} 1^{\mathfrak{F}(\mathrm{Src}\, f)}\right)\right)^{-1} = \left(I^{\mathsf{RLD}}_{\mathcal{A}}\circ f^{-1}\circ I^{\mathsf{RLD}}_{\mathcal{B}}\right)^{-1} = I^{\mathsf{RLD}}_{\mathcal{B}}\circ f\circ I^{\mathsf{RLD}}_{\mathcal{A}}$.
□