Theorem 44 *For every small sets* A*,* B*,* C *if* $g, h \in \mathsf{RLD}(A; B)$ *then*

1. $f \circ (g \cup h) = f \circ g \cup f \circ h$ *for every* $f \in \mathsf{RLD}(B; C)$*;*
2. $(g \cup h) \circ f = g \circ f \cup h \circ f$ *for every* $f \in \mathsf{RLD}(C; A)$*.*

Proof We'll prove only the first as the second is dual.

By the infinite distributivity law for filters we have

$$\begin{aligned} f \circ g \cup f \circ h &= \bigcap \left\{ \uparrow^{\mathsf{RLD}(A;C)} (F \circ G) \mid F \in \operatorname{up} f, G \in \operatorname{up} g \right\} \cup \bigcap \left\{ \uparrow^{\mathsf{RLD}(A;C)} (F \circ H) \mid F \in \operatorname{up} f, H \in \operatorname{up} h \right\} \\ &= \bigcap \left\{ \uparrow^{\mathsf{RLD}(A;C)} (F_1 \circ G) \cup \uparrow^{\mathsf{RLD}(A;C)} (F_2 \circ H) \mid F_1, F_2 \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h \right\} \\ &= \bigcap \left\{ \uparrow^{\mathsf{RLD}(A;C)} ((F_1 \circ G) \cup (F_2 \circ H)) \mid F_1, F_2 \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h \right\}. \end{aligned}$$

Obviously

$$\begin{aligned} \bigcap \left\{ \uparrow^{\mathsf{RLD}(A;C)} ((F_1 \circ G) \cup (F_2 \circ H)) \mid F_1, F_2 \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h \right\} &\supseteq \\ \bigcap \left\{ \uparrow^{\mathsf{RLD}(A;C)} (((F_1 \cap F_2) \circ G) \cup ((F_1 \cap F_2) \circ H)) \mid F_1, F_2 \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h \right\} &= \\ \bigcap \left\{ \uparrow^{\mathsf{RLD}(A;C)} ((F \circ G) \cup (F \circ H)) \mid F \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h \right\} &= \\ \bigcap \left\{ \uparrow^{\mathsf{RLD}(A;C)} (F \circ (G \cup H)) \mid F \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h \right\}. \end{aligned}$$

Because $G \in \operatorname{up} g \wedge H \in \operatorname{up} h \Rightarrow G \cup H \in \operatorname{up}(g \cup h)$ we have

$$\begin{aligned} \bigcap \left\{ \uparrow^{\mathsf{RLD}(A;C)} (F \circ (G \cup H)) \mid F \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h \right\} &\supseteq \\ \bigcap \left\{ \uparrow^{\mathsf{RLD}(A;C)} (F \circ K) \mid F \in \operatorname{up} f, K \in \operatorname{up}(g \cup h) \right\} &= \\ f \circ (g \cup h). \end{aligned}$$

Thus we proved $f \circ g \cup f \circ h \supseteq f \circ (g \cup h)$. But obviously $f \circ (g \cup h) \supseteq f \circ g$ and $f \circ (g \cup h) \supseteq f \circ h$ and so $f \circ (g \cup h) \supseteq f \circ g \cup f \circ h$. Thus $f \circ (g \cup h) = f \circ g \cup f \circ h$.

□

Conjecture 7 *If* f *and* g *are reloids, then*

$$g \circ f = \bigcup \left\{ G \circ F \mid F \in \operatorname{atoms} f, G \in \operatorname{atoms} g \right\}.$$

Theorem 45 *Let* A*,* B*,* C *be sets,* $f \in \mathsf{RLD}(A; B)$*,* $g \in \mathsf{RLD}(B; C)$*,* $h \in \mathsf{RLD}(A; C)$*. Then*

$$g \circ f \not\asymp h \Leftrightarrow g \not\asymp h \circ f^{-1}.$$

Proof $g \circ f \not\asymp h \Leftrightarrow \bigcap \left\{ \uparrow^{\mathsf{RLD}(A;C)} (G \circ F) \mid F \in \operatorname{up} f, G \in \operatorname{up} g \right\} \cap \bigcap \left\langle \uparrow^{\mathsf{RLD}(A;C)} \right\rangle \operatorname{up} h \neq 0^{\mathsf{RLD}(A;C)} \Leftrightarrow \bigcap \left\{ \left\langle \uparrow^{\mathsf{RLD}(A;C)} \right\rangle ((G \circ F) \cap H) \mid F \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h \right\} \neq 0^{\mathsf{RLD}(A;C)} \Leftrightarrow \forall F \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h : \uparrow^{\mathsf{RLD}(A;C)} ((G \circ F) \cap H) \neq$