Proof Let denote R the right part of the equality to prove.

$\langle R\rangle^* \{\beta\} = \bigcup\left\{ \langle f|_{\uparrow^{\operatorname{Src} f}\{\alpha\}}\rangle^* \{\beta\} \quad | \quad \alpha \in \operatorname{Src} f \right\} = \langle f\rangle^* \{\beta\}$ for every $\beta \in \operatorname{Src} f$ and R is complete as a join of complete funcoids.

Thus R is the completion of f. □

Conjecture 6 $\operatorname{Compl} f = f \setminus^* (\Omega \times^{\mathsf{FCD}} \mho)$ *for every funcoid* f*.*

This conjecture may be proved by considerations similar to these in the section "Fréchet filter" in [15].

Lemma 2 *Co-completion of a complete funcoid is complete.*

Proof Let f is a complete funcoid.

$\langle \operatorname{CoCompl} f\rangle^* X = \operatorname{Cor} \langle f\rangle^* X = \operatorname{Cor} \bigcup \left\{ \langle f\rangle^* \{x\} \quad | \quad x \in X \right\} = \bigcup \left\{ \operatorname{Cor} \langle f\rangle^* \{x\} \quad | \quad x \in X \right\} = \bigcup \left\{ \langle \operatorname{CoCompl} f\rangle^* \{x\} \quad | \quad x \in X \right\}$ for every set X. Thus $\operatorname{CoCompl} f$ is complete. □

Theorem 40 $\operatorname{Compl} \operatorname{CoCompl} f = \operatorname{CoCompl} \operatorname{Compl} f = \operatorname{Cor} f$ *for every funcoid* f*.*

Proof $\operatorname{Compl} \operatorname{CoCompl} f$ is co-complete since (used the lemma) $\operatorname{CoCompl} f$ is co-complete. Thus $\operatorname{Compl} \operatorname{CoCompl} f$ is a principal funcoid. $\operatorname{CoCompl} f$ is the the greatest co-complete funcoid under f and $\operatorname{Compl} \operatorname{CoCompl} f$ is the greatest complete funcoid under $\operatorname{CoCompl} f$. So $\operatorname{Compl} \operatorname{CoCompl} f$ is greater than any principal funcoid under $\operatorname{CoCompl} f$ which is greater than any principal funcoid under f. Thus $\operatorname{Compl} \operatorname{CoCompl} f$ it is the greatest principal funcoid under f. Thus $\operatorname{Compl} \operatorname{CoCompl} f = \operatorname{Cor} f$. Similarly $\operatorname{CoCompl} \operatorname{Compl} f = \operatorname{Cor} f$. □

Question 16. Is $\operatorname{Compl} \mathsf{FCD}(A; B)$ a co-brouwerian lattice for every small sets A, B?

3.13 Monovalued and injective funcoids

Following the idea of definition of monovalued morphism let's call **monovalued** such a funcoid f that $f \circ f^{-1} \subseteq I^{\mathsf{FCD}}_{\operatorname{im} f}$.

Similarly, I will call a funcoid **injective** when $f^{-1} \circ f \subseteq I^{\mathsf{FCD}}_{\operatorname{dom} f}$.

Obvious 17. A funcoid f is

- monovalued iff $f \circ f^{-1} \subseteq I^{\mathsf{FCD}(\operatorname{Dst} f)}$;
- injective iff $f^{-1} \circ f \subseteq I^{\mathsf{FCD}(\operatorname{Src} f)}$.

In other words, a funcoid is monovalued (injective) when it is a monovalued (injective) morphism of the category of funcoids.

Monovaluedness is dual of injectivity.

Obvious 18.