**Theorem 36** $\langle \operatorname{CoCompl} f\rangle^* X = \operatorname{Cor}\langle f\rangle^* X$ *for every funcoid* $f$ *and set* $X \in \mathscr{P}(\operatorname{Src} f)$.

**Proof** $\operatorname{CoCompl} f \subseteq f$ thus $\langle \operatorname{CoCompl} f\rangle^* X \subseteq \langle f\rangle^* X$, but $\langle \operatorname{CoCompl} f\rangle^* X$ is a principal f.o. thus $\langle \operatorname{CoCompl} f\rangle^* X \subseteq \operatorname{Cor}\langle f\rangle^* X$.

Let $\alpha X = \operatorname{Cor}\langle f\rangle^* X$. Then $\alpha\emptyset = 0^{\mathfrak{F}(\operatorname{Dst} f)}$ and

$$\alpha(X\cup Y) = \operatorname{Cor}\langle f\rangle^*(X\cup Y) = \operatorname{Cor}(\langle f\rangle^* X\cup\langle f\rangle^* Y) = \operatorname{Cor}\langle f\rangle^* X\cup\operatorname{Cor}\langle f\rangle^* Y = \alpha X\cup\alpha Y.$$

(used the theorem 65 from [15]). Thus $\alpha$ can be continued till $\langle g\rangle$ for some funcoid $g$. This funcoid is co-complete.

Evidently $g$ is the greatest co-complete element of $\mathsf{FCD}(\operatorname{Src} f; \operatorname{Dst} f)$ which is lower than $f$.

Thus $g = \operatorname{CoCompl} f$ and so $\operatorname{Cor}\langle f\rangle^* X = \alpha X = \langle g\rangle^* X = \langle \operatorname{CoCompl} f\rangle^* X$. □

**Theorem 37** $\operatorname{Compl}\mathsf{FCD}(A;B)$ *is an atomistic lattice.*

**Proof** Let $f \in \operatorname{Compl}\mathsf{FCD}(A;B)$. $\langle f\rangle^* X = \bigcup\left\{\langle f\rangle^*\{x\} \mid x \in X\right\} = \bigcup\left\{\left\langle f|_{\uparrow^{\operatorname{Src} f}\{x\}}\right\rangle^*\{x\} \mid x \in X\right\} = \bigcup\left\{\left\langle f|_{\uparrow^{\operatorname{Src} f}\{x\}}\right\rangle^* X \mid x \in X\right\}$, thus $f = \bigcup\left\{f|_{\uparrow^{\operatorname{Src} f}\{x\}} \mid x \in X\right\}$. It is trivial that every $f|_{\uparrow^{\operatorname{Src} f}\{x\}}$ is a join of atoms of $\operatorname{Compl}\mathsf{FCD}(A;B)$. □

**Theorem 38** *A funcoid* $f$ *is complete iff it is a join (on the lattice* $\mathsf{FCD}(\operatorname{Src} f; \operatorname{Dst} f)$*) of atomic complete funcoids.*

**Proof** It follows from the theorem 29 and the previous theorem. □

**Corollary 11** $\operatorname{Compl}\mathsf{FCD}(A;B)$ *is join-closed.*

**Theorem 39** $\operatorname{Compl}(\bigcup R) = \bigcup\langle\operatorname{Compl}\rangle R$ *for every* $R \in \mathscr{P}\mathsf{FCD}(A;B)$ *(for every small sets* $A$, $B$*).*

**Proof** $\langle \operatorname{Compl}(\bigcup R)\rangle^* X = \bigcup\left\{\langle\bigcup R\rangle^*\{\alpha\} \mid \alpha \in X\right\} = \bigcup\left\{\bigcup\left\{\langle f\rangle^*\{\alpha\} \mid f \in R\right\} \mid \alpha \in X\right\} = \bigcup\left\{\bigcup\left\{\langle f\rangle^*\{\alpha\} \mid \alpha \in X\right\} \mid f \in R\right\} = \bigcup\left\{\langle \operatorname{Compl} f\rangle^* X \mid f \in R\right\} = \langle\bigcup\langle\operatorname{Compl}\rangle R\rangle^* X$ for every set $X$. □

**Corollary 12** Compl *is a lower adjoint.*

**Conjecture 5** Compl *is not an upper adjoint (in general).*

**Proposition 25** $\operatorname{Compl} f = \bigcup\left\{f|_{\uparrow^{\operatorname{Src} f}\{\alpha\}} \mid \alpha \in \operatorname{Src} f\right\}$ *for every funcoid* $f$.