**Proof** We will prove only the first as the second is similar. Let

$$f = \bigcup \left\{ \uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{FCD}} F(\alpha) \quad | \quad \alpha \in \operatorname{Src} f \right\} = \bigcup \left\{ \uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{FCD}} G(\alpha) \quad | \quad \alpha \in \operatorname{Src} f \right\}$$

for some $F, G \in \mathfrak{F}(\operatorname{Dst} f)^{\operatorname{Src} f}$. We need to prove $F = G$. Let $\beta \in \operatorname{Src} f$.

$$\langle f \rangle \{\beta\} = \bigcup \left\{ \left\langle \uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{FCD}} F(\alpha) \right\rangle \{\beta\} \quad | \quad \alpha \in \operatorname{Src} f \right\} = F(\beta).$$

Similarly $\langle f \rangle \{\beta\} = G(\beta)$. So $F(\beta) = G(\beta)$. □

## 3.12 Completion of funcoids

**Theorem 34** $\operatorname{Cor} f = \operatorname{Cor}' f$ *for an element* $f$ *of a filtrator of funcoids. (Core part is taken for the shifted filtrator of funcoids.)*

**Proof** From the theorem 26 in [15] and the corollary 10 and theorem 30. □

**Definition 35** ***Completion*** *of a funcoid* $f \in \mathsf{FCD}(A; B)$ *is the complete funcoid* $\operatorname{Compl} f \in \mathsf{FCD}(A; B)$ *defined by the formula* $\langle \operatorname{Compl} f \rangle^* \{\alpha\} = \langle f \rangle^* \{\alpha\}$ *for* $\alpha \in \operatorname{Src} f$.

**Definition 36** ***Co-completion*** *of a funcoid* $f$ *is defined by the formula*

$$\operatorname{CoCompl} f = (\operatorname{Compl} f^{-1})^{-1}.$$

**Obvious 15.** $\operatorname{Compl} f \subseteq f$ and $\operatorname{CoCompl} f \subseteq f$ for every funcoid $f$.

**Proposition 24** *The filtrator* $(\mathsf{FCD}(A; B); \operatorname{Compl} \mathsf{FCD}(A; B))$ *is filtered.*

**Proof** Because the shifted filtrator $(\mathsf{FCD}(A; B); \mathscr{P}(A \times B); \uparrow^{\mathsf{FCD}(A;B)})$ is filtered. □

**Theorem 35** $\operatorname{Compl} f = \operatorname{Cor}^{(\mathsf{FCD}(A;B);\operatorname{Compl} \mathsf{FCD}(A;B))} f = \operatorname{Cor}'^{(\mathsf{FCD}(A;B);\operatorname{Compl} \mathsf{FCD}(A;B))} f$ *for every funcoid* $f \in \mathsf{FCD}(A; B)$.

**Proof** $\operatorname{Cor}^{(\mathsf{FCD}(A;B);\operatorname{Compl} \mathsf{FCD}(A;B))} f = \operatorname{Cor}'^{(\mathsf{FCD}(A;B);\operatorname{Compl} \mathsf{FCD}(A;B))} f$ since (the theorem 26 in [15]) the filtrator $(\mathsf{FCD}(A; B); \operatorname{Compl} \mathsf{FCD}(A; B))$ is filtered and with join closed core (the theorem 29).

Let $g \in \operatorname{up}^{(\mathsf{FCD}(A;B);\operatorname{Compl} \mathsf{FCD}(A;B))} f$. Then $g \in \operatorname{Compl} \mathsf{FCD}(A; B)$ and $g \supseteq f$. Thus $g = \operatorname{Compl} g \supseteq \operatorname{Compl} f$.

Thus $\forall g \in \operatorname{up}^{(\mathsf{FCD}(A;B);\operatorname{Compl} \mathsf{FCD}(A;B))} f : g \supseteq \operatorname{Compl} f$.

Let $\forall g \in \operatorname{up}^{(\mathsf{FCD}(A;B);\operatorname{Compl} \mathsf{FCD}(A;B))} f : h \subseteq g$ for some $h \in \operatorname{Compl} \mathsf{FCD}(A; B)$.

Then $h \subseteq \bigcap^{\mathsf{FCD}(A;B)} \operatorname{up}^{(\mathsf{FCD}(A;B);\operatorname{Compl} \mathsf{FCD}(A;B))} f = f$ and consequently $h = \operatorname{Compl} h \subseteq \operatorname{Compl} f$.

Thus

$$\operatorname{Compl} f = \bigcap^{\operatorname{Compl} \mathsf{FCD}(A;B)} \operatorname{up}^{(\mathsf{FCD}(A;B);\operatorname{Compl} \mathsf{FCD}(A;B))} f = \operatorname{Cor}^{(\mathsf{FCD}(A;B);\operatorname{Compl} \mathsf{FCD}(A;B))} f.$$

□