Proof First, it's easy to see that $\{\alpha\} \times^{\mathsf{FCD}} b$ are elements of $\operatorname{Compl} \mathsf{FCD}(A;B)$. Also $0^{\mathsf{FCD}(A;B)}$ is an element of $\operatorname{Compl} \mathsf{FCD}(A;B)$.

$\uparrow^A \{\alpha\} \times^{\mathsf{FCD}} b$ are atoms of $\operatorname{Compl} \mathsf{FCD}(A;B)$ because these are atoms of $\mathsf{FCD}(A;B)$.

It remains to prove that if f is an atom of $\operatorname{Compl} \mathsf{FCD}(A;B)$ then $f = \{\alpha\} \times^{\mathsf{FCD}} b$ for some $\alpha \in A$ and an atomic f.o. b on B.

Suppose $f \in \mathsf{FCD}(A;B)$ is a non-empty complete funcoid. Then exists $\alpha \in A$ such that $\langle f \rangle^* \{\alpha\} \neq 0^{\mathfrak{F}(B)}$. Thus $\uparrow^A \{\alpha\} \times^{\mathsf{FCD}} b \subseteq f$ for some atomic f.o. b on B. If f is an atom then $f = \uparrow^A \{\alpha\} \times^{\mathsf{FCD}} b$. □

Theorem 32

1. *A funcoid* $f \in \mathsf{FCD}(A;B)$ *is complete iff there exists a function* $G : A \to \mathfrak{F}(B)$ *such that*

$$f = \bigcup \left\{ \uparrow^A \{\alpha\} \times^{\mathsf{FCD}} G(\alpha) \quad | \quad \alpha \in A \right\}. \tag{11}$$

2. *A funcoid* $f \in \mathsf{FCD}(A;B)$ *is co-complete iff there exists a function* $G : B \to \mathfrak{F}(A)$ *such that*

$$f = \bigcup \left\{ G(\alpha) \times^{\mathsf{FCD}} \uparrow^B \{\alpha\} \quad | \quad \alpha \in B \right\}.$$

Proof We will prove only the first as the second is symmetric.

⇒ Let f is complete. Then take

$$G(\alpha) = \bigcup \left\{ b \in \operatorname{atoms} 1^{\mathfrak{F}(\operatorname{Dst} f)} \quad | \quad \uparrow^A \{\alpha\} \times^{\mathsf{FCD}} b \subseteq f \right\}$$

and we have (11) obviously.

⇐ Let (11) holds. Then $G(\alpha) = \bigcup \operatorname{atoms} G(\alpha)$ and thus

$$f = \bigcup \left\{ \uparrow^A \{\alpha\} \times^{\mathsf{FCD}} b \quad | \quad \alpha \in \operatorname{Src} f, b \in \operatorname{atoms} G(\alpha) \right\}$$

and so f is complete.

□

Theorem 33

1. *For a complete funcoid* f *there exists exactly one function* $F \in \mathfrak{F}(\operatorname{Dst} f)^{\operatorname{Src} f}$ *such that*

$$f = \bigcup \left\{ \uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{FCD}} F(\alpha) \quad | \quad \alpha \in \operatorname{Src} f \right\}.$$

2. *For a co-complete funcoid* f *there exists exactly one function* $F \in \mathfrak{F}(\operatorname{Src} f)^{\operatorname{Dst} f}$ *such that*

$$f = \bigcup \left\{ F(\alpha) \times^{\mathsf{FCD}} \uparrow^{\operatorname{Dst} f} \{\alpha\} \quad | \quad \alpha \in \operatorname{Dst} f \right\}.$$