Proof It's enough to prove that every funcoid is representable as an (infinite) meet (on the lattice $\mathsf{FCD}(A;B)$) of some set of principal funcoids.

Let $f \in \mathsf{FCD}(A;B)$, $X \in \mathscr{P}A$, $Y \in \operatorname{up}\langle f\rangle X$, $g(X;Y) \overset{\text{def}}{=} \uparrow^A X \times^{\mathsf{FCD}} \uparrow^B Y \cup \uparrow^A \overline{X} \times^{\mathsf{FCD}} 1^{\mathfrak{F}(B)}$. For every $K \in \mathscr{P}A$

$$\langle g(X;Y)\rangle^* K = \left\langle \uparrow^A X \times^{\mathsf{FCD}} \uparrow^B Y\right\rangle^* K \cup \left\langle \uparrow^A \overline{X} \times^{\mathsf{FCD}} 1^{\mathfrak{F}(B)}\right\rangle^* K = \left(\begin{cases} 0^{\mathfrak{F}(B)} & \text{if } K = \emptyset \\ Y & \text{if } \emptyset \neq K \subseteq X \\ 1^{\mathfrak{F}(B)} & \text{if } K \nsubseteq X \end{cases}\right) \supseteq \langle f\rangle^* K;$$

so $g(X;Y) \supseteq f$. For every $X \in \mathscr{P}A$

$$\bigcap \left\{ \langle g(X;Y)\rangle^* X \quad | \quad Y \in \operatorname{up}\langle f\rangle X \right\} = \bigcap \left\{ Y \quad | \quad Y \in \operatorname{up}\langle f\rangle^* X \right\} = \langle f\rangle^* X;$$

consequently

$$\left\langle \bigcap \left\{ g(X;Y) \quad | \quad X \in \mathscr{P}A, Y \in \operatorname{up}\langle f\rangle^* X \right\} \right\rangle^* X \subseteq \langle f\rangle^* X$$

that is

$$\bigcap \left\{ g(X;Y) \quad | \quad X \in \mathscr{P}A, Y \in \operatorname{up}\langle f\rangle^* X \right\} \subseteq f$$

and finally

$$f = \bigcap \left\{ g(X;Y) \quad | \quad X \in \mathscr{P}A, Y \in \operatorname{up}\langle f\rangle^* X \right\}.$$

□

Conjecture 3 *If $f \in \mathsf{FCD}(B;C)$ is a complete funcoid and $R \in \mathscr{P}\mathsf{FCD}(A;B)$ then $f \circ \bigcup R = \bigcup \langle f\circ\rangle R$.*

This conjecture can be weakened:

Conjecture 4 *If f is a principal funcoid from B to C and $R \in \mathscr{P}\mathsf{FCD}(A;B)$ then $f \circ \bigcup R = \bigcup \langle f\circ\rangle R$.*

I will denote $\operatorname{Compl}\mathsf{FCD}$ and $\operatorname{CoCompl}\mathsf{FCD}$ the sets of complete and co-complete funcoids correspondingly. $\operatorname{Compl}\mathsf{FCD}(A;B)$ are complete funcoids from A to B and likewise with $\operatorname{CoCompl}\mathsf{FCD}(A;B)$.

Obvious 14. $\operatorname{Compl}\mathsf{FCD}$ and $\operatorname{CoCompl}\mathsf{FCD}$ are closed regarding composition of funcoids.

Proposition 23 $\operatorname{Compl}\mathsf{FCD}(A;B)$ *and* $\operatorname{CoCompl}\mathsf{FCD}(A;B)$ *(with induced order) are complete lattices.*

Proof It follows from the theorem 29. □

Theorem 31 *Atoms of the lattice* $\operatorname{Compl}\mathsf{FCD}(A;B)$ *are exactly funcoidal products of the form $\uparrow^A \{\alpha\} \times^{\mathsf{FCD}} b$ where $\alpha \in A$ and b is an atomic f.o. on B*