Obvious 13. A funcoid f is co-complete iff $\langle f\rangle^* = \uparrow^{\mathrm{Dst}\, f} \circ \alpha$ for a generalized closure α.

Remark 4 Thus funcoids can be considered as a generalization of generalized closures. A topological space in Kuratowski sense is the same as reflexive and transitive generalized closure. So topological spaces can be considered as a special case of funcoids.

Definition 34 *I will call a* ***complete funcoid*** *a funcoid whose reverse is co-complete.*

Theorem 27 *The following conditions are equivalent for every funcoid f:*

1. *funcoid f is complete;*
2. $\forall S \in \mathscr{P}\mathfrak{F}(\mathrm{Src}\, f), J \in \mathscr{P}(\mathrm{Dst}\, f) : (\bigcup S\ [f] \uparrow^{\mathrm{Dst}\, f} J \Leftrightarrow \exists \mathcal{I} \in S : \mathcal{I}\ [f] \uparrow^{\mathrm{Dst}\, f} J)$;
3. $\forall S \in \mathscr{P}\mathscr{P}(\mathrm{Src}\, f), J \in \mathscr{P}(\mathrm{Dst}\, f) : (\bigcup S\ [f]^* J \Leftrightarrow \exists I \in S : I\ [f]^* J)$;
4. $\forall S \in \mathscr{P}\mathfrak{F}(\mathrm{Src}\, f) : \langle f\rangle \bigcup S = \bigcup \langle\langle f\rangle\rangle S$;
5. $\forall S \in \mathscr{P}\mathscr{P}(\mathrm{Src}\, f) : \langle f\rangle^* \bigcup S = \bigcup \left\langle \langle f\rangle^* \right\rangle S$;
6. $\forall A \in \mathscr{P}(\mathrm{Src}\, f) : \langle f\rangle^* A = \bigcup \left\{ \langle f\rangle^* \{a\} \mid a \in A \right\}$.

Proof

(3)⇒(1) For every $S \in \mathscr{P}\mathscr{P}(\mathrm{Src}\, f)$, $J \in \mathscr{P}(\mathrm{Dst}\, f)$

$$\uparrow^{\mathrm{Src}\, f} \bigcup S \cap \left\langle f^{-1} \right\rangle^* J \neq 0^{\mathfrak{F}(\mathrm{Src}\, f)} \Leftrightarrow \exists I \in S : \uparrow^{\mathrm{Src}\, f} I \cap \left\langle f^{-1} \right\rangle^* J \neq 0^{\mathfrak{F}(\mathrm{Src}\, f)}, \tag{10}$$

consequently by the theorem 53 in [15] we have that $\left\langle f^{-1} \right\rangle^* J$ is a principal f.o.

(1)⇒(2) For every $S \in \mathscr{P}\mathfrak{F}(\mathrm{Src}\, f)$, $J \in \mathscr{P}(\mathrm{Dst}\, f)$ we have $\left\langle f^{-1} \right\rangle^* J$ a principal f.o., consequently

$$\bigcup S \cap \left\langle f^{-1} \right\rangle^* J \neq 0^{\mathfrak{F}(\mathrm{Src}\, f)} \Leftrightarrow \exists \mathcal{I} \in S : \mathcal{I} \cap \left\langle f^{-1} \right\rangle^* J \neq 0^{\mathfrak{F}(\mathrm{Src}\, f)}.$$

From this follows (2).

(6)⇒(5) $\langle f\rangle^* \bigcup S = \bigcup \left\{ \langle f\rangle^* \{a\} \mid a \in \bigcup S \right\} = \bigcup \left\{ \bigcup \left\{ \langle f\rangle^* \{a\} \mid a \in A \right\} \mid A \in S \right\} = \bigcup \left\{ \langle f\rangle^* A \mid A \in S \right\} = \bigcup \left\langle \langle f\rangle^* \right\rangle S$.

(2)⇒(4) $\uparrow^{\mathrm{Dst}\, f} J \not\asymp \langle f\rangle \bigcup S \Leftrightarrow \bigcup S\ [f] \uparrow^{\mathrm{Dst}\, f} J \Leftrightarrow \exists \mathcal{I} \in S : \mathcal{I}\ [f] \uparrow^{\mathrm{Dst}\, f} J \Leftrightarrow \exists \mathcal{I} \in S : \uparrow^{\mathrm{Dst}\, f} J \not\asymp \langle f\rangle \mathcal{I} \Leftrightarrow \uparrow^{\mathrm{Dst}\, f} J \not\asymp \bigcup \langle\langle f\rangle\rangle S$ (used the theorem 53 in [15]).