$\square$

Note that distributivity of the lattice of funcoids is proved through using atoms of this lattice. I have never seen such method of proving distributivity.

Corollary 9 *The lattice* $\mathsf{FCD}(A;B)$ *is co-brouwerian (for every small sets* A *and* B*).*

The next proposition is one more (among the theorem 12) generalization for funcoids of composition of relations.

Proposition 21 *For every composable funcoids* f, g

$$\operatorname{atoms}(g\circ f) = \left\{ x\times^{\mathsf{FCD}} z \;\middle|\; \begin{array}{c} x\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Src}f)}, z\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Dst}g)},\\ \exists y\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Dst}f)} : (x\times^{\mathsf{FCD}} y\in\operatorname{atoms}f\wedge y\times^{\mathsf{FCD}} z\in\operatorname{atoms}g)\end{array}\right\}.$$

Proof $(x\times^{\mathsf{FCD}} z)\cap(g\circ f)\neq 0\Leftrightarrow x\;[g\circ f]\;z\Leftrightarrow\exists y\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Dst}f)}:(x\;[f]\;y\wedge y\;[g]\;z)\Leftrightarrow\exists y\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Dst}f)}:((x\times^{\mathsf{FCD}} y)\cap f\neq 0\wedge(y\times^{\mathsf{FCD}} z)\cap g\neq 0)$ (it was used the theorem 12). $\square$

Theorem 26 *Let* f *be a funcoid.*

1. $\mathcal{X}\;[f]\;\mathcal{Y}\Leftrightarrow\exists F\in\operatorname{atoms}f:\mathcal{X}\;[F]\;\mathcal{Y}$;
2. $\langle f\rangle\mathcal{X}=\bigsqcup_{F\in\operatorname{atoms}f}\langle F\rangle\mathcal{X}$ *for every* $\mathcal{X}\in\mathfrak{F}(\operatorname{Src}f)$.

Proof 1. $\exists F\in\operatorname{atoms}f:\mathcal{X}\;[F]\;\mathcal{Y}\Leftrightarrow\exists a\in\mathfrak{F}(\operatorname{Src}f),b\in\mathfrak{F}(\operatorname{Dst}f):\left(a\times^{\mathrm{FCD}} b\not\asymp f\wedge\mathcal{X}\;[a\times^{\mathrm{FCD}} b]\;\mathcal{Y}\right)\Leftrightarrow\exists a\in\mathfrak{F}(\operatorname{Src}f),b\in\mathfrak{F}(\operatorname{Dst}f):(a\times^{\mathrm{FCD}} b\not\asymp f\wedge a\times^{\mathrm{FCD}} b\not\asymp\mathcal{X}\times^{\mathrm{FCD}}\mathcal{Y})\Leftrightarrow\exists F\in\operatorname{atoms}f:(F\not\asymp f\wedge F\not\asymp\mathcal{X}\times^{\mathrm{FCD}}\mathcal{Y})\Leftrightarrow f\not\asymp\mathcal{X}\times^{\mathrm{FCD}}\mathcal{Y}\Leftrightarrow\mathcal{X}\;[f]\;\mathcal{Y}$.

2. Let $\mathcal{Y}\in\mathfrak{F}(\operatorname{Dst}f)$. Suppose $\mathcal{Y}\not\asymp\langle f\rangle\mathcal{X}$. Then $\mathcal{X}\;[f]\;\mathcal{Y}$; $\exists F\in\operatorname{atoms}f:\mathcal{X}\;[F]\;\mathcal{Y}$; $\exists F\in\operatorname{atoms}f:\mathcal{Y}\not\asymp\langle F\rangle\mathcal{X}$; $\mathcal{Y}\not\asymp\bigsqcup_{F\in\operatorname{atoms}f}\langle F\rangle\mathcal{X}$. So $\langle f\rangle\mathcal{X}\sqsubseteq\bigsqcup_{F\in\operatorname{atoms}f}\langle F\rangle\mathcal{X}$. The contrary $\langle f\rangle\mathcal{X}\sqsupseteq\bigsqcup_{F\in\operatorname{atoms}f}\langle F\rangle\mathcal{X}$ is obvious. $\square$

3.11 Complete funcoids

Definition 33 *I will call* ***co-complete*** *such a funcoid* f *that* $\langle f\rangle^{*}X$ *is a principal f.o. for every* $X\in\mathscr{P}(\operatorname{Src}f)$.

Remark 3 I will call **generalized closure** such a function $\alpha\in\mathscr{P}B^{\mathscr{P}A}$ (for some small sets A, B) that

1. $\alpha\emptyset=\emptyset$;
2. $\forall I,J\in\mathscr{P}A:\alpha(I\cup J)=\alpha I\cup\alpha J$.