So

$$\left\langle \bigcap \left\{ \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \quad | \quad (\mathcal{A}; \mathcal{B}) \in S \right\} \right\rangle x = \begin{cases} \bigcap \operatorname{im} S & \text{if} \quad x \not\asymp \bigcap \operatorname{dom} S; \\ 0^{\mathfrak{F}(B)} & \text{if} \quad x \asymp \bigcap \operatorname{dom} S. \end{cases}$$

From this follows the statement of the theorem. □

Corollary 6 *For every $\mathcal{A}_0, \mathcal{A}_1 \in \mathfrak{F}(A)$, $\mathcal{B}_0, \mathcal{B}_1 \in \mathfrak{F}(B)$ (for every small sets A, B)*

$$(\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0) \cap (\mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1) = (\mathcal{A}_0 \cap \mathcal{A}_1) \times^{\mathsf{FCD}} (\mathcal{B}_0 \cap \mathcal{B}_1).$$

Proof $(\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0) \cap (\mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1) = \bigcap \left\{ \mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0, \mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1 \right\}$ what is by the last theorem equal to $(\mathcal{A}_0 \cap \mathcal{A}_1) \times^{\mathsf{FCD}} (\mathcal{B}_0 \cap \mathcal{B}_1)$. □

Theorem 21 *If A, B are small sets and $\mathcal{A} \in \mathfrak{F}(A)$* then *$\mathcal{A}\times^{\mathsf{FCD}}$ is a complete homomorphism from the lattice $\mathfrak{F}(B)$ to the lattice $\mathsf{FCD}(A; B)$, if also $\mathcal{A} \neq 0^{\mathfrak{F}(A)}$ then it is an order embedding.*

Proof Let $S \in \mathscr{P}\mathfrak{F}(B)$, $X \in \mathscr{P}A$, $x \in \operatorname{atoms} 1^{\mathfrak{F}(A)}$.

$$\begin{aligned} \left\langle \bigcup \left\langle \mathcal{A}\times^{\mathsf{FCD}} \right\rangle S \right\rangle^* X &= \bigcup \left\{ \left\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \right\rangle^* X \quad | \quad \mathcal{B} \in S \right\} \\ &= \begin{cases} \bigcup S & \text{if } X \in \partial \mathcal{A} \\ 0^{\mathfrak{F}(B)} & \text{if } X \notin \partial \mathcal{A} \end{cases} \\ &= \left\langle \mathcal{A} \times^{\mathsf{FCD}} \bigcup S \right\rangle^* X; \end{aligned}$$

$$\begin{aligned} \left\langle \bigcap \left\langle \mathcal{A}\times^{\mathsf{FCD}} \right\rangle S \right\rangle x &= \bigcap \left\{ \left\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \right\rangle x \quad | \quad \mathcal{B} \in S \right\} \\ &= \begin{cases} \bigcap S & \text{if } x \not\asymp \mathcal{A} \\ 0^{\mathfrak{F}(B)} & \text{if } x \asymp \mathcal{A} \end{cases} \\ &= \left\langle \mathcal{A} \times^{\mathsf{FCD}} \bigcap S \right\rangle x. \end{aligned}$$

Thus $\bigcup \left\langle \mathcal{A}\times^{\mathsf{FCD}} \right\rangle S = \mathcal{A} \times^{\mathsf{FCD}} \bigcup S$ and $\bigcap \left\langle \mathcal{A}\times^{\mathsf{FCD}} \right\rangle S = \mathcal{A} \times^{\mathsf{FCD}} \bigcap S$.

If $\mathcal{A} \neq 0^{\mathfrak{F}(A)}$ then obviously the function $\mathcal{A}\times^{\mathsf{FCD}}$ is injective. □

The following proposition states that cutting a rectangle of atomic width from a funcoid always produces a rectangular (representable as a funcoidal product of filter objects) funcoid (of atomic width).

Proposition 19 *If $f \in \mathsf{FCD}$ and a is an atomic filter object on* $\operatorname{Src} f$ *then*

$$f|_a = a \times^{\mathsf{FCD}} \langle f \rangle a.$$

Proof Let $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$.

$$\mathcal{X} \not\asymp a \Rightarrow \langle f|_a \rangle \mathcal{X} = \langle f \rangle a, \quad \mathcal{X} \asymp a \Rightarrow \langle f|_a \rangle \mathcal{X} = 0^{\mathfrak{F}(\operatorname{Dst} f)}.$$

□