Theorem 19 $f \cap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) = I^{\mathsf{FCD}}_{\mathcal{B}} \circ f \circ I^{\mathsf{FCD}}_{\mathcal{A}}$ *for every funcoid* f *and* $\mathcal{A} \in \mathfrak{F}(\operatorname{Src} f)$, $\mathcal{B} \in \mathfrak{F}(\operatorname{Dst} f)$.

Proof $h \overset{\text{def}}{=} I^{\mathsf{FCD}}_{\mathcal{B}} \circ f \circ I^{\mathsf{FCD}}_{\mathcal{A}}$. For every $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$

$$\langle h \rangle \mathcal{X} = \left\langle I^{\mathsf{FCD}}_{\mathcal{B}} \right\rangle \langle f \rangle \left\langle I^{\mathsf{FCD}}_{\mathcal{A}} \right\rangle \mathcal{X} = \mathcal{B} \cap \langle f \rangle (\mathcal{A} \cap \mathcal{X}).$$

From this, as easy to show, $h \subseteq f$ and $h \subseteq \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}$. If $g \subseteq f \wedge g \subseteq \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}$ for a $g \in \mathsf{FCD}(\operatorname{Src} f; \operatorname{Dst} f)$ then $\operatorname{dom} g \subseteq \mathcal{A}$, $\operatorname{im} g \subseteq \mathcal{B}$,

$$\langle g \rangle \mathcal{X} = \mathcal{B} \cap \langle g \rangle (\mathcal{A} \cap \mathcal{X}) \subseteq \mathcal{B} \cap \langle f \rangle (\mathcal{A} \cap \mathcal{X}) = \left\langle I^{\mathsf{FCD}}_{\mathcal{B}} \right\rangle \langle f \rangle \left\langle I^{\mathsf{FCD}}_{\mathcal{A}} \right\rangle \mathcal{X} = \langle h \rangle \mathcal{X},$$

$g \subseteq h$. So $h = f \cap^{\mathsf{FCD}} (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B})$. □

Corollary 3 $f|_{\mathcal{A}} = f \cap (\mathcal{A} \times^{\mathsf{FCD}} 1^{\mathfrak{F}(\operatorname{Dst} f)})$ *for every* $f \in \mathsf{FCD}$ *and* $\mathcal{A} \in \mathfrak{F}(\operatorname{Src} f)$.

Proof $f \cap (\mathcal{A} \times^{\mathsf{FCD}} 1^{\mathfrak{F}(\operatorname{Dst} f)}) = I^{\mathsf{FCD}}_{1^{\mathfrak{F}(\operatorname{Dst} f)}} \circ f \circ I^{\mathsf{FCD}}_{\mathcal{A}} = f \circ I^{\mathsf{FCD}}_{\mathcal{A}} = f|_{\mathcal{A}}$. □

Corollary 4 $f \not\asymp (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) \Leftrightarrow \mathcal{A}\ [f]\ \mathcal{B}$ *for every* $f \in \mathsf{FCD}$, $\mathcal{A} \in \mathfrak{F}(\operatorname{Src} f)$, $\mathcal{B} \in \mathfrak{F}(\operatorname{Dst} f)$.

Proof $f \not\asymp (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) \Leftrightarrow \left\langle f \cap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) \right\rangle^{*} (\operatorname{Src} f) \neq 0^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow \left\langle I^{\mathsf{FCD}}_{\mathcal{B}} \circ f \circ I^{\mathsf{FCD}}_{\mathcal{A}} \right\rangle^{*} (\operatorname{Src} f) \neq 0^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow \left\langle I^{\mathsf{FCD}}_{\mathcal{B}} \right\rangle \langle f \rangle \left\langle I^{\mathsf{FCD}}_{\mathcal{A}} \right\rangle 1^{\mathfrak{F}(\operatorname{Src} f)} \neq 0^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow \mathcal{B} \cap \langle f \rangle (\mathcal{A} \cap 1^{\mathfrak{F}(\operatorname{Src} f)}) \neq 0^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow \mathcal{B} \cap \langle f \rangle \mathcal{A} \neq 0^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow \mathcal{A}\ [f]\ \mathcal{B}$. □

Corollary 5 *Every filtrator of funcoids is star-separable.*

Proof The set of funcoidal products of principal filter objects is a separation subset of the lattice of funcoids. □

Theorem 20 *Let* A, B *be small sets. If* $S \in \mathscr{P}(\mathfrak{F}(A) \times \mathfrak{F}(B))$ *then*

$$\bigcap \left\{ \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \quad | \quad (\mathcal{A}; \mathcal{B}) \in S \right\} = \bigcap \operatorname{dom} S \times^{\mathsf{FCD}} \bigcap \operatorname{im} S.$$

Proof If $x \in \operatorname{atoms} 1^{\mathfrak{F}(A)}$ then by the theorem 17

$$\left\langle \bigcap \left\{ \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \quad | \quad (\mathcal{A}; \mathcal{B}) \in S \right\} \right\rangle x = \bigcap \left\{ \left\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \right\rangle x \quad | \quad (\mathcal{A}; \mathcal{B}) \in S \right\}.$$

If $x \not\asymp \bigcap \operatorname{dom} S$ then

$$\forall (\mathcal{A}; \mathcal{B}) \in S : (x \cap \mathcal{A} \neq 0^{\mathfrak{F}(A)} \wedge \left\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \right\rangle x = \mathcal{B});$$
$$\left\{ \left\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \right\rangle x \quad | \quad (\mathcal{A}; \mathcal{B}) \in S \right\} = \operatorname{im} S;$$

if $x \asymp \bigcap \operatorname{dom} S$ then

$$\exists (\mathcal{A}; \mathcal{B}) \in S : \left(x \cap \mathcal{A} = 0^{\mathfrak{F}(A)} \wedge \left\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \right\rangle x = 0^{\mathfrak{F}(B)} \right);$$
$$\left\{ \left\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \right\rangle x \quad | \quad (\mathcal{A}; \mathcal{B}) \in S \right\} \ni 0^{\mathfrak{F}(B)}.$$