Obviously $\neg(X\ \delta'\ \emptyset)$ and $\neg(\emptyset\ \delta'\ Y)$.

For suitable I and J we have:

$$\begin{aligned}(I \cup J)\ \delta'\ Y &\Leftrightarrow \exists x \in \operatorname{atoms} \uparrow^A (I \cup J), y \in \operatorname{atoms} \uparrow^B Y : x\ \delta\ y\\ &\Leftrightarrow \exists x \in \operatorname{atoms} \uparrow^A I \cup \operatorname{atoms} \uparrow^A J, y \in \operatorname{atoms} \uparrow^B Y : x\ \delta\ y\\ &\Leftrightarrow \exists x \in \operatorname{atoms} \uparrow^A I, y \in \operatorname{atoms} \uparrow^B Y : x\ \delta\ y \vee \exists x \in \operatorname{atoms} \uparrow^A J, y \in \operatorname{atoms} \uparrow^B Y : x\ \delta\ y\\ &\Leftrightarrow I\ \delta'\ Y \vee J\ \delta'\ Y;\end{aligned}$$

similarly $X\ \delta'\ (I \cup J) \Leftrightarrow X\ \delta'\ I \vee X\ \delta'\ J$ for suitable I and J. Let's continue δ' till a funcoid f (by the theorem 7):

$$\mathcal{X}\ [f]\ \mathcal{Y} \Leftrightarrow \forall X \in \operatorname{up} \mathcal{X}, Y \in \operatorname{up} \mathcal{Y} : X\ \delta'\ Y.$$

The reverse of (8) implication is trivial, so

$$\forall X \in \operatorname{up} a, Y \in \operatorname{up} b \exists x \in \operatorname{atoms} \uparrow^A X, y \in \operatorname{atoms} \uparrow^B Y : x\ \delta\ y \Leftrightarrow a\ \delta\ b.$$

$\forall X \in \operatorname{up} a, Y \in \operatorname{up} b \exists x \in \operatorname{atoms} \uparrow^A X, y \in \operatorname{atoms} \uparrow^B Y : x\ \delta\ y \Leftrightarrow \forall X \in \operatorname{up} a, Y \in \operatorname{up} b : X\ \delta'\ Y \Leftrightarrow a\ [f]\ b$.

So $a\ \delta\ b \Leftrightarrow a\ [f]\ b$, that is $[f]$ is a continuation of δ. □

One of uses of the previous theorem is the proof of the following theorem:

Theorem 17 *If A, B are small sets, $R \in \mathscr{P}\mathsf{FCD}(A; B)$, $x \in \operatorname{atoms} 1^{\mathfrak{F}(A)}$, $y \in \operatorname{atoms} 1^{\mathfrak{F}(B)}$, then*

1. $\langle \bigcap R \rangle x = \bigcap \{ \langle f \rangle x \mid f \in R \}$;

2. $x\ [\bigcap R]\ y \Leftrightarrow \forall f \in R : x\ [f]\ y$.

Proof 2. Let denote $x\ \delta\ y \Leftrightarrow \forall f \in R : x\ [f]\ y$. For every $a \in \operatorname{atoms} 1^{\mathfrak{F}(A)}$, $b \in \operatorname{atoms} 1^{\mathfrak{F}(B)}$

$$\begin{gathered}\forall X \in \operatorname{up} a, Y \in \operatorname{up} b \exists x \in \operatorname{atoms} \uparrow^A X, y \in \operatorname{atoms} \uparrow^B Y : x\ \delta\ y \Rightarrow\\ \forall f \in R, X \in \operatorname{up} a, Y \in \operatorname{up} b \exists x \in \operatorname{atoms} \uparrow^A X, y \in \operatorname{atoms} \uparrow^B Y : x\ [f]\ y \Rightarrow\\ \forall f \in R, X \in \operatorname{up} a, Y \in \operatorname{up} b : X\ [f]^*\ Y \Rightarrow\\ \forall f \in R : a\ [f]\ b \Leftrightarrow\\ a\ \delta\ b.\end{gathered}$$

So, by the theorem 16, δ can be continued till $[p]$ for some funcoid $p \in \mathsf{FCD}(A; B)$.

For every funcoid $q \in \mathsf{FCD}(A; B)$ such that $\forall f \in R : q \subseteq f$ we have $x\ [q]\ y \Rightarrow \forall f \in R : x\ [f]\ y \Leftrightarrow x\ \delta\ y \Leftrightarrow x\ [p]\ y$, so $q \subseteq p$. Consequently $p = \bigcap R$.

From this $x\ [\bigcap R]\ y \Leftrightarrow \forall f \in R : x\ [f]\ y$.

1. From the former $y \in \operatorname{atoms} \langle \bigcap R \rangle x \Leftrightarrow y \cap \langle \bigcap R \rangle x \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \forall f \in R : y \cap \langle f \rangle x \neq 0^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow y \in \bigcap \langle \operatorname{atoms} \rangle \{ \langle f \rangle x \mid f \in R \} \Leftrightarrow y \in \operatorname{atoms} \bigcap \{ \langle f \rangle x \mid f \in R \}$ for every $y \in \operatorname{atoms} 1^{\mathfrak{F}(A)}$. From this follows $\langle \bigcap R \rangle x = \bigcap \{ \langle f \rangle x \mid f \in R \}$. □