2. *A relation* $\delta \in \mathscr{P}(\operatorname{atoms} 1^{\mathfrak{F}(A)} \times \operatorname{atoms} 1^{\mathfrak{F}(B)})$ *such that (for every* $a \in \operatorname{atoms} 1^{\mathfrak{F}(A)}$, $b \in \operatorname{atoms} 1^{\mathfrak{F}(B)}$*)*

$$\forall X \in \operatorname{up} a, Y \in \operatorname{up} b \exists x \in \operatorname{atoms} {\uparrow^A} X, y \in \operatorname{atoms} {\uparrow^B} Y : x \ \delta \ y \Rightarrow a \ \delta \ b \quad (8)$$

can be continued to the relation $[f]$ *for a unique* $f \in \mathsf{FCD}(A; B)$*;*

$$\mathcal{X} \ [f] \ \mathcal{Y} \Leftrightarrow \exists x \in \operatorname{atoms} \mathcal{X}, y \in \operatorname{atoms} \mathcal{Y} : x \ \delta \ y \quad (9)$$

for every $\mathcal{X} \in \mathfrak{F}(A)$, $\mathcal{Y} \in \mathfrak{F}(B)$.

Proof Existence of no more than one such funcoids and formulas (7) and (9) follow from the previous theorem.

1. Consider the function $\alpha' \in \mathfrak{F}(B)^{\mathscr{P}A}$ defined by the formula (for every $X \in \mathscr{P}A$)

$$\alpha' X = \bigcup \langle \alpha \rangle \operatorname{atoms} {\uparrow^A} X.$$

Obviously $\alpha' \emptyset = 0^{\mathfrak{F}(B)}$. For every $I, J \in \mathscr{P}A$

$$\begin{aligned} \alpha'(I \cup J) &= \bigcup \langle \alpha \rangle \operatorname{atoms} {\uparrow^A} (I \cup J) \\ &= \bigcup \langle \alpha \rangle (\operatorname{atoms} {\uparrow^A} I \cup \operatorname{atoms} {\uparrow^A} J) \\ &= \bigcup \left(\langle \alpha \rangle \operatorname{atoms} {\uparrow^A} I \cup \langle \alpha \rangle \operatorname{atoms} {\uparrow^A} J \right) \\ &= \bigcup \langle \alpha \rangle \operatorname{atoms} {\uparrow^A} I \cup \bigcup \langle \alpha \rangle \operatorname{atoms} {\uparrow^A} J. \\ &= \alpha' I \cup \alpha' J. \end{aligned}$$

Let continue α' till a funcoid f (by the theorem 7): $\langle f \rangle \mathcal{X} = \bigcap \langle \alpha' \rangle \operatorname{up} \mathcal{X}$.

Let's prove the reverse of (6):

$$\begin{aligned} \bigcap \left\langle \bigcup \circ \langle \alpha \rangle \circ \operatorname{atoms} \circ {\uparrow^A} \right\rangle \operatorname{up} a &= \bigcap \left\langle \bigcup \circ \langle \alpha \rangle \right\rangle \langle \operatorname{atoms} \rangle \langle {\uparrow^A} \rangle \operatorname{up} a \\ &\subseteq \bigcap \left\langle \bigcup \circ \langle \alpha \rangle \right\rangle \{\{a\}\} \\ &= \bigcap \left\{ \left(\bigcup \circ \langle \alpha \rangle \right) \{a\} \right\} \\ &= \bigcap \left\{ \bigcup \langle \alpha \rangle \{a\} \right\} \\ &= \bigcap \left\{ \bigcup \{\alpha a\} \right\} = \bigcap \{\alpha a\} = \alpha a. \end{aligned}$$

Finally,

$$\alpha a = \bigcap \left\langle \bigcup \circ \langle \alpha \rangle \circ \operatorname{atoms} \circ {\uparrow^A} \right\rangle \operatorname{up} a = \bigcap \langle \alpha' \rangle \operatorname{up} a = \langle f \rangle a,$$

so $\langle f \rangle$ is a continuation of α.

2. Consider the relation $\delta' \in \mathscr{P}(\mathscr{P}A \times \mathscr{P}B)$ defined by the formula (for every $X \in \mathscr{P}A$, $Y \in \mathscr{P}B$)

$$X \ \delta' \ Y \Leftrightarrow \exists x \in \operatorname{atoms} {\uparrow^A} X, y \in \operatorname{atoms} {\uparrow^B} Y : x \ \delta \ y.$$