To show it is really a category is trivial.

The **category of funcoid triples** is defined as follows:

- Objects are filter objects on small sets.
- The morphisms from a f.o. $\mathcal{A}$ to a f.o. $\mathcal{B}$ are triples $(f;\mathcal{A};\mathcal{B})$ where $f \in \mathsf{FCD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{B}))$ and $\mathrm{dom}\, f \subseteq \mathcal{A} \wedge \mathrm{im}\, f \subseteq \mathcal{B}$.
- The composition is defined by the formula $(g;\mathcal{B};\mathcal{C}) \circ (f;\mathcal{A};\mathcal{B}) = (g \circ f;\mathcal{A};\mathcal{C})$.
- Identity morphism for an f.o. $\mathcal{A}$ is $I^{\mathsf{FCD}}_{\mathcal{A}}$.

To prove that it is really a category is trivial.

3.8 Specifying funcoids by functions or relations on atomic filter objects

Theorem 15 *For every funcoid f and $\mathcal{X} \in \mathfrak{F}(\mathrm{Src}\, f)$, $\mathcal{Y} \in \mathfrak{F}(\mathrm{Dst}\, f)$*

1. $\langle f \rangle \mathcal{X} = \bigcup \langle\langle f \rangle\rangle \mathrm{atoms}\, \mathcal{X}$;
2. $\mathcal{X} \; [f] \; \mathcal{Y} \Leftrightarrow \exists x \in \mathrm{atoms}\, \mathcal{X}, y \in \mathrm{atoms}\, \mathcal{Y} : x \; [f] \; y$.

Proof 1.

$$\begin{aligned}
\mathcal{Y} \cap \langle f \rangle \mathcal{X} \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)} \quad &\Leftrightarrow \quad \mathcal{X} \cap \left\langle f^{-1} \right\rangle \mathcal{Y} \neq 0^{\mathfrak{F}(\mathrm{Src}\, f)} \\
&\Leftrightarrow \quad \exists x \in \mathrm{atoms}\, \mathcal{X} : x \cap \left\langle f^{-1} \right\rangle \mathcal{Y} \neq 0^{\mathfrak{F}(\mathrm{Src}\, f)} \\
&\Leftrightarrow \quad \exists x \in \mathrm{atoms}\, \mathcal{X} : \mathcal{Y} \cap \langle f \rangle x \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)}.
\end{aligned}$$

$\partial \langle f \rangle \mathcal{X} = \bigcup \langle \partial \rangle \langle\langle f \rangle\rangle \mathrm{atoms}\, \mathcal{X} = \partial \bigcup \langle\langle f \rangle\rangle \mathrm{atoms}\, \mathcal{X}$.

2. If $\mathcal{X} \; [f] \; \mathcal{Y}$, then $\mathcal{Y} \cap \langle f \rangle \mathcal{X} \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)}$, consequently exists $y \in \mathrm{atoms}\, \mathcal{Y}$ such that $y \cap \langle f \rangle \mathcal{X} \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)}$, $\mathcal{X} \; [f] \; y$. Repeating this second time we get that there exists $x \in \mathrm{atoms}\, \mathcal{X}$ such that $x \; [f] \; y$. From this follows

$$\exists x \in \mathrm{atoms}\, \mathcal{X}, y \in \mathrm{atoms}\, \mathcal{Y} : x \; [f] \; y.$$

The reverse is obvious. □

Theorem 16 *Let A and B be small sets.*

1. *A function $\alpha \in \mathfrak{F}(B)^{\mathrm{atoms}\, 1^{\mathfrak{F}(A)}}$ such that (for every $a \in \mathrm{atoms}\, 1^{\mathfrak{F}(A)}$)*

$$\alpha a \subseteq \bigcap \left\langle \bigcup \circ \langle \alpha \rangle \circ \mathrm{atoms} \circ \uparrow^{A} \right\rangle \mathrm{up}\, a \tag{6}$$

can be continued to the function $\langle f \rangle$ for a unique $f \in \mathsf{FCD}(A;B)$;

$$\langle f \rangle \mathcal{X} = \bigcup \langle \alpha \rangle \mathrm{atoms}\, \mathcal{X} \tag{7}$$

for every $\mathcal{X} \in \mathfrak{F}(A)$.