Proof $\mathcal{X} \cap \operatorname{dom} f \neq 0^{\mathfrak{F}(\operatorname{Src} f)} \Leftrightarrow \mathcal{X} \cap \left\langle f^{-1} \right\rangle 1^{\mathfrak{F}(\operatorname{Dst} f)} \neq 0^{\mathfrak{F}(\operatorname{Src} f)} \Leftrightarrow 1^{\mathfrak{F}(\operatorname{Dst} f)} \cap \langle f \rangle \mathcal{X} \neq 0^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow \langle f \rangle \mathcal{X} \neq 0^{\mathfrak{F}(\operatorname{Dst} f)}$. □

Corollary 2 $\operatorname{dom} f = \bigcup \left\{ a \in \operatorname{atoms} 1^{\mathfrak{F}(\operatorname{Src} f)} \mid \langle f \rangle a \neq 0^{\mathfrak{F}(\operatorname{Dst} f)} \right\}$.

Proof This follows from the fact that $\mathfrak{F}(\operatorname{Src} f)$ is an atomistic lattice. □

Proposition 15 $\operatorname{dom} f|_{\mathcal{A}} = \mathcal{A} \cap \operatorname{dom} f$ *for every funcoid* f *and* $\mathcal{A} \in \mathfrak{F}(\operatorname{Src} f)$.

Proof $\operatorname{dom} f|_{\mathcal{A}} = \operatorname{im}\left(I^{\mathsf{FCD}}_{\mathcal{A}} \circ f^{-1}\right) = \left\langle I^{\mathsf{FCD}}_{\mathcal{A}} \right\rangle \left\langle f^{-1} \right\rangle 1^{(\operatorname{Dst} f)} = \mathcal{A} \cap \left\langle f^{-1} \right\rangle 1^{(\operatorname{Dst} f)} = \mathcal{A} \cap \operatorname{dom} f$. □

Theorem 14 $\operatorname{im} f = \bigcap \left\langle \uparrow^{\operatorname{Dst} f} \right\rangle \langle \operatorname{im} \rangle \operatorname{up} f$ *and* $\operatorname{dom} f = \bigcap \left\langle \uparrow^{\operatorname{Src} f} \right\rangle \langle \operatorname{dom} \rangle \operatorname{up} f$ *for every funcoid* f.

Proof $\operatorname{im} f = \langle f \rangle 1^{\mathfrak{F}(\operatorname{Src} f)} = \bigcap \left\{ \left\langle \uparrow^{\mathsf{FCD}(\operatorname{Src} f; \operatorname{Dst} f)} F \right\rangle 1^{\mathfrak{F}(\operatorname{Src} f)} \mid F \in \operatorname{up} f \right\} = \bigcap \left\{ \uparrow^{\operatorname{Dst} f} \operatorname{im} F \mid F \in \operatorname{up} f \right\} = \bigcap \left\langle \uparrow^{\operatorname{Dst} f} \right\rangle \langle \operatorname{im} \rangle \operatorname{up} f$ (used the theorem 9).

The second formula follows from symmetry. □

Proposition 16 *For every composable funcoids* f, g:

1. *If* $\operatorname{im} f \supseteq \operatorname{dom} g$ *then* $\operatorname{im}(g \circ f) = \operatorname{im} g$.
2. *If* $\operatorname{im} f \subseteq \operatorname{dom} g$ *then* $\operatorname{dom}(g \circ f) = \operatorname{dom} f$.

Proof

1. $\operatorname{im}(g \circ f) = \langle g \circ f \rangle 1^{\mathfrak{F}(\operatorname{Src} f)} = \langle g \rangle \langle f \rangle 1^{\mathfrak{F}(\operatorname{Src} f)} = \langle g \rangle \operatorname{im} f = \langle g \rangle (\operatorname{im} f \cap \operatorname{dom} g) = \langle g \rangle \operatorname{dom} g = \langle g \rangle 1^{\mathfrak{F}(\operatorname{Src} g)} = \operatorname{im} g$.
2. $\operatorname{dom}(g \circ f) = \operatorname{im}\left(f^{-1} \circ g^{-1}\right)$ what by proved above is equal to $\operatorname{im} f^{-1}$ that is $\operatorname{dom} f$.

□

3.7 Categories of funcoids

I will define two categories, the **category of funcoids** and the **category of funcoid triples**.

The **category of funcoids** is defined as follows:

- Objects are small sets.
- The set of morphisms from a set A to a set B is $\mathsf{FCD}(A; B)$.
- The composition is the composition of funcoids.
- Identity morphism for a set is the identity funcoid for that set.