So $\langle h\rangle \circ \uparrow^A = \alpha$ for some funcoid h. Obviously

$$\forall f \in R : h \supseteq f. \tag{5}$$

And h is the least funcoid for which holds the condition (5). So $h = \bigcup R$.

1 $X\ [\bigcup R]^*\ Y \Leftrightarrow \uparrow^{\mathrm{Dst}\, f} Y \cap \langle \bigcup R\rangle^* X \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)} \Leftrightarrow \uparrow^{\mathrm{Dst}\, f} Y \cap \bigcup \left\{ \langle f\rangle^* X \mid f \in R \right\} \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)} \Leftrightarrow \exists f \in R : \uparrow^{\mathrm{Dst}\, f} Y \cap \langle f\rangle^* X \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)} \Leftrightarrow \exists f \in R : X\ [f]^*\ Y$ (used the theorem 40 in [15]).

□

In the next theorem, compared to the previous one, the class of infinite unions is replaced with lesser class of finite unions and simultaneously class of sets is changed to more wide class of filter objects.

Theorem 11 *For every $f, g \in \mathsf{FCD}(A; B)$ and $\mathcal{X} \in \mathfrak{F}(A)$ (for every small sets A, B)*

1. $\langle f \cup g\rangle \mathcal{X} = \langle f\rangle \mathcal{X} \cup \langle g\rangle \mathcal{X}$;
2. $[f \cup g] = [f] \cup [g]$.

Proof

1. Let $\alpha\mathcal{X} \stackrel{\mathrm{def}}{=} \langle f\rangle \mathcal{X} \cup \langle g\rangle \mathcal{X}$; $\beta\mathcal{Y} \stackrel{\mathrm{def}}{=} \left\langle f^{-1}\right\rangle \mathcal{Y} \cup \left\langle g^{-1}\right\rangle \mathcal{Y}$ for every $\mathcal{X} \in \mathfrak{F}(A)$, $\mathcal{Y} \in \mathfrak{F}(B)$. Then

$$\begin{aligned} \mathcal{Y} \cap \alpha\mathcal{X} \neq 0^{\mathfrak{F}(B)} &\Leftrightarrow \mathcal{Y} \cap \langle f\rangle \mathcal{X} \neq 0^{\mathfrak{F}(B)} \vee \mathcal{Y} \cap \langle g\rangle \mathcal{X} \neq 0^{\mathfrak{F}(B)} \\ &\Leftrightarrow \mathcal{X} \cap \left\langle f^{-1}\right\rangle \mathcal{Y} \neq 0^{\mathfrak{F}(A)} \vee \mathcal{X} \cap \left\langle g^{-1}\right\rangle \mathcal{Y} \neq 0^{\mathfrak{F}(A)} \\ &\Leftrightarrow \mathcal{X} \cap \beta\mathcal{Y} \neq 0^{\mathfrak{F}(A)}. \end{aligned}$$

So $h = (A; B; \alpha; \beta)$ is a funcoid. Obviously $h \supseteq f$ and $h \supseteq g$. If $p \supseteq f$ and $p \supseteq g$ for some funcoid p then $\langle p\rangle \mathcal{X} \supseteq \langle f\rangle \mathcal{X} \cup \langle g\rangle \mathcal{X} = \langle h\rangle \mathcal{X}$ that is $p \supseteq h$. So $f \cup g = h$.

2. $\mathcal{X}\ [f \cup g]\ \mathcal{Y} \Leftrightarrow \mathcal{Y} \cap \langle f \cup g\rangle \mathcal{X} \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \mathcal{Y} \cap (\langle f\rangle \mathcal{X} \cup \langle g\rangle \mathcal{X}) \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \mathcal{Y} \cap \langle f\rangle \mathcal{X} \neq 0^{\mathfrak{F}(B)} \vee \mathcal{Y} \cap \langle g\rangle \mathcal{X} \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \mathcal{X}\ [f]\ \mathcal{Y} \vee \mathcal{X}\ [g]\ \mathcal{Y}$ for every $\mathcal{X} \in \mathfrak{F}(A)$, $\mathcal{Y} \in \mathfrak{F}(B)$.

□

3.5 More on composition of funcoids

Proposition 11 $[g \circ f] = [g] \circ \langle f\rangle = \left\langle g^{-1}\right\rangle^{-1} \circ [f]$ *for every composable funcoids f and g.*