Lemma 1 *$\langle f\rangle^* X = \bigcap\left\{\uparrow^{\operatorname{Dst} f}\langle F\rangle X \mid F\in\operatorname{up} f\right\}$ for every funcoid f and set $X\in\mathscr{P}(\operatorname{Src} f)$.*

Proof Obviously $\langle f\rangle^* X \subseteq \bigcap\left\{\uparrow^{\operatorname{Dst} f}\langle F\rangle X \mid F\in\operatorname{up} f\right\}$.

Let $B\in\operatorname{up}\langle f\rangle^* X$. Let $F_B = X\times B\cup((\operatorname{Src} f)\setminus X)\times(\operatorname{Dst} f)$.

$\langle F_B\rangle X = B$.

We have $\emptyset\neq P\subseteq X\Rightarrow\langle F_B\rangle P = B\supseteq\langle f\rangle^* P$ and $\emptyset\neq P\nsubseteq X\Rightarrow\langle F_B\rangle P = \operatorname{Dst} f\supseteq\langle f\rangle^* P$. Thus $\langle F_B\rangle P\supseteq\langle f\rangle^* P$ for every set $P\in\mathscr{P}(\operatorname{Src} f)$ and so $\uparrow^{\mathsf{FCD}(\operatorname{Src} f;\operatorname{Dst} f)} F_B\supseteq f$ that is $F_B\in\operatorname{up} f$.

Thus $\forall B\in\operatorname{up}\langle f\rangle^* X : B\in\operatorname{up}\bigcap\left\{\uparrow^{\operatorname{Dst} f}\langle F\rangle X \mid F\in\operatorname{up} f\right\}$ because $B\in\operatorname{up}\uparrow^{\operatorname{Dst} f}\langle F_B\rangle X$.

So $\bigcap\left\{\uparrow^{\operatorname{Dst} f}\langle F\rangle X \mid F\in\operatorname{up} f\right\}\subseteq\langle f\rangle^* X$. □

Theorem 9 *$\langle f\rangle\mathcal{X} = \bigcap\left\{\left\langle\uparrow^{\mathsf{FCD}(\operatorname{Src} f;\operatorname{Dst} f)} F\right\rangle\mathcal{X} \mid F\in\operatorname{up} f\right\}$ for every funcoid f and $\mathcal{X}\in\mathfrak{F}(\operatorname{Src} f)$.*

Proof $\bigcap\left\{\left\langle\uparrow^{\mathsf{FCD}(\operatorname{Src} f;\operatorname{Dst} f)} F\right\rangle\mathcal{X} \mid F\in\operatorname{up} f\right\} = \bigcap\left\{\bigcap\left\langle\uparrow^{\operatorname{Dst} f}\right\rangle\langle\langle F\rangle\rangle\operatorname{up}\mathcal{X} \mid F\in\operatorname{up} f\right\} = \bigcap\left\{\bigcap\left\{\uparrow^{\operatorname{Dst} f}\langle F\rangle X \mid X\in\operatorname{up}\mathcal{X}\right\} \mid F\in\operatorname{up} f\right\} = \bigcap\left\{\bigcap\left\{\uparrow^{\operatorname{Dst} f}\langle F\rangle X \mid F\in\operatorname{up} f\right\} \mid X\in\operatorname{up}\mathcal{X}\right\} = \bigcap\left\{\uparrow^{\operatorname{Dst} f}\langle f\rangle^* X \mid X\in\operatorname{up}\mathcal{X}\right\} = \langle f\rangle\mathcal{X}$ (the lemma used). □

Conjecture 1 *Every filtrator of funcoids is:*

1. *with separable core;*
2. *with co-separable core.*

Below it is shown that $\mathsf{FCD}(A;B)$ are complete lattices for every small sets A and B. We will apply lattice operations to subsets of such sets without explicitly mentioning $\mathsf{FCD}(A;B)$.

Theorem 10 *$\mathsf{FCD}(A;B)$ is a complete lattice (for every small sets A and B). For every $R\in\mathscr{P}\mathsf{FCD}(A;B)$ and $X\in\mathscr{P}A$, $Y\in\mathscr{P}B$*

1. *$X\left[\bigcup R\right]^* Y\Leftrightarrow\exists f\in R : X[f]^* Y$;*
2. *$\left\langle\bigcup R\right\rangle^* X = \bigcup\left\{\langle f\rangle^* X \mid f\in R\right\}$.*

Proof Accordingly [14] to prove that it is a complete lattice it's enough to prove existence of all joins.

2 $\alpha X\stackrel{\text{def}}{=}\bigcup\left\{\langle f\rangle^* X \mid f\in R\right\}$. We have $\alpha\emptyset = 0^{\mathfrak{F}(\operatorname{Dst} f)}$;

$$\begin{aligned}\alpha(I\cup J) &= \bigcup\left\{\langle f\rangle^*(I\cup J) \mid f\in R\right\}\\ &= \bigcup\left\{\langle f\rangle^* I\cup\langle f\rangle^* J \mid f\in R\right\}\\ &= \bigcup\left\{\langle f\rangle^* I \mid f\in R\right\}\cup\bigcup\left\{\langle f\rangle^* J \mid f\in R\right\}\\ &= \alpha I\cup\alpha J.\end{aligned}$$