That is the formulas (3) are true.

Accordingly the above there exists a funcoid f such that

$$\mathcal{X}\ [f]\ \mathcal{Y} \Leftrightarrow \forall X \in \operatorname{up}\mathcal{X}, Y \in \operatorname{up}\mathcal{Y} : X\ \delta\ Y.$$

$\forall X \in \mathscr{P}A, Y \in \mathscr{P}B : \left(\uparrow^B Y \cap \langle f\rangle \uparrow^A X \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \uparrow^A X\ [f] \uparrow^B Y \Leftrightarrow X\ \delta\ Y \Leftrightarrow \uparrow^B Y \cap \alpha X \neq 0^{\mathfrak{F}(B)}\right)$, consequently $\forall X \in \mathscr{P}A : \alpha X = \langle f\rangle \uparrow^A X = \langle f\rangle^* X$. □

Note that by the last theorem to every proximity δ corresponds a unique funcoid. So funcoids are a generalization of (quasi-)proximity structures.

Reverse funcoids can be considered as a generalization of conjugate quasi-proximity.

Definition 24 *Any small (multivalued) function $F : A \to B$ corresponds to a funcoid $\uparrow^{\mathsf{FCD}(A;B)} F \in \mathsf{FCD}(A;B)$, where by definition $\left\langle \uparrow^{\mathsf{FCD}(A;B)} F\right\rangle \mathcal{X} = \bigcap \left\langle \uparrow^B \right\rangle \langle\langle F\rangle\rangle \operatorname{up}\mathcal{X}$ for every $\mathcal{X} \in \mathfrak{F}(A)$.*

Using the last theorem it is easy to show that this definition is monovalued and does not contradict to former stuff. (Take $\alpha = \uparrow^B \circ \langle F\rangle$.)

Definition 25 *Funcoids corresponding to a binary relation (= multivalued function) are called **principal funcoids**.*

We may equate principal funcoids with corresponding binary relations by the method of appendix B in [15]. This is useful for describing relationships of funcoids and binary relations, such as for the formulas of continuous functions and continuous funcoids (see below).

Theorem 8 *If S is a generalized filter base on $\operatorname{Src} f$ then $\langle f\rangle \bigcap S = \bigcap \langle\langle f\rangle\rangle S$ for every funcoid f.*

Proof $\langle f\rangle \bigcap S \subseteq \langle f\rangle X$ for every $X \in S$ and thus $\langle f\rangle \bigcap S \subseteq \bigcap \langle\langle f\rangle\rangle S$.

By properties of generalized filter bases:

$\langle f\rangle \bigcap S = \bigcap \left\langle \langle f\rangle^* \right\rangle \operatorname{up} \bigcap S =$
$\bigcap \left\langle \langle f\rangle^* \right\rangle \{X \mid \exists \mathcal{P} \in S : X \in \operatorname{up}\mathcal{P}\} = \bigcap \left\{\langle f\rangle^* X \mid \exists \mathcal{P} \in S : X \in \operatorname{up}\mathcal{P}\right\} \supseteq$
$\bigcap \{\langle f\rangle \mathcal{P} \mid \mathcal{P} \in S\} = \bigcap \langle\langle f\rangle\rangle S$. □

3.4 Lattices of funcoids

Definition 26 $f \subseteq g \overset{\text{def}}{=} [f] \subseteq [g]$ *for $f, g \in \mathsf{FCD}$.*

Thus every $\mathsf{FCD}(A;B)$ is a poset. (It's taken into account that $[f] \neq [g]$ if $f \neq g$.)

Definition 27 *I will call a **shifted filtrator of funcoids** the shifted filtrator*

$$\left(\mathsf{FCD}(A;B); \mathscr{P}(A \times B); \uparrow^{\mathsf{FCD}(A;B)}\right)$$

for some small sets A, B.

$\operatorname{up} f \overset{\text{def}}{=} \operatorname{up}^{\left(\mathsf{FCD}(A;B); \mathscr{P}(A\times B); \uparrow^{\mathsf{FCD}(A;B)}\right)} f$ *for every funcoid $f \in \mathsf{FCD}(A;B)$.*