For such δ it holds (for every $\mathcal{X} \in \mathfrak{F}(A)$, $\mathcal{Y} \in \mathfrak{F}(B)$)

$$\mathcal{X} \left[L_R^{-1}\delta\right] \mathcal{Y} \Leftrightarrow \forall X \in \operatorname{up}\mathcal{X}, Y \in \operatorname{up}\mathcal{Y} : X \;\delta\; Y. \tag{4}$$

Proof Injectivity of L_F and L_R, formulas (2) (for $\alpha \in \operatorname{im} L_F$) and (4) (for $\delta \in \operatorname{im} L_R$), formulas (1) and (3) follow from two previous theorems. The only thing remained to prove is that for every α and δ that obey the above conditions a corresponding funcoid f exists.

2. Let define $\alpha \in \mathfrak{F}(B)^{\mathscr{P}A}$ by the formula $\partial(\alpha X) = \{Y \in \mathscr{P}B \mid X \;\delta\; Y\}$ for every $X \in \mathscr{P}A$. (It is obvious that $\{Y \in \mathscr{P}B \mid X \;\delta\; Y\}$ is a free star.) Analogously it can be defined $\beta \in \mathfrak{F}(A)^{\mathscr{P}B}$ by the formula $\partial(\beta Y) = \{X \in \mathscr{P}A \mid X \;\delta\; Y\}$. Let's continue α and β to $\alpha' \in \mathfrak{F}(B)^{\mathfrak{F}(A)}$ and $\beta' \in \mathfrak{F}(A)^{\mathfrak{F}(B)}$ by the formulas

$$\alpha'\mathcal{X} = \bigcap \langle\alpha\rangle \operatorname{up}\mathcal{X} \quad \text{and} \quad \beta'\mathcal{Y} = \bigcap \langle\beta\rangle \operatorname{up}\mathcal{Y}$$

and δ to $\delta' \in \mathscr{P}(\mathfrak{F}(A) \times \mathfrak{F}(B))$ by the formula

$$\mathcal{X} \;\delta'\; \mathcal{Y} \Leftrightarrow \forall X \in \operatorname{up}\mathcal{X}, Y \in \operatorname{up}\mathcal{Y} : X \;\delta\; Y.$$

$\mathcal{Y} \cap \alpha'\mathcal{X} \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \mathcal{Y} \cap \bigcap \langle\alpha\rangle \operatorname{up}\mathcal{X} \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \bigcap \langle\mathcal{Y}\cap\rangle \langle\alpha\rangle \operatorname{up}\mathcal{X} \neq 0^{\mathfrak{F}(B)}$. Let's prove that

$$W = \langle\mathcal{Y}\cap\rangle \langle\alpha\rangle \operatorname{up}\mathcal{X}$$

is a generalized filter base: To prove it is enough to show that $\langle\alpha\rangle \operatorname{up}\mathcal{X}$ is a generalized filter base. If $\mathcal{A}, \mathcal{B} \in \langle\alpha\rangle \operatorname{up}\mathcal{X}$ then exist $X_1, X_2 \in \operatorname{up}\mathcal{X}$ such that $\mathcal{A} = \alpha X_1$ and $\mathcal{B} = \alpha X_2$.

Then $\alpha(X_1 \cap X_2) \in \langle\alpha\rangle \operatorname{up}\mathcal{X}$. So $\langle\alpha\rangle \operatorname{up}\mathcal{X}$ is a generalized filter base and thus W is a generalized filter base.

Accordingly to the corollary 1 of the theorem 1, $\bigcap \langle\mathcal{Y}\cap\rangle \langle\alpha\rangle \operatorname{up}\mathcal{X} \neq 0^{\mathfrak{F}(B)}$ is equivalent to

$$\forall X \in \operatorname{up}\mathcal{X} : \mathcal{Y} \cap \alpha X \neq 0^{\mathfrak{F}(B)},$$

what is equivalent to $\forall X \in \operatorname{up}\mathcal{X}, Y \in \operatorname{up}\mathcal{Y} : \uparrow^B Y \cap \alpha X \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \forall X \in \operatorname{up}\mathcal{X}, Y \in \operatorname{up}\mathcal{Y} : Y \in \partial(\alpha X) \Leftrightarrow \forall X \in \operatorname{up}\mathcal{X}, Y \in \operatorname{up}\mathcal{Y} : X \;\delta\; Y$. Combining the equivalencies we get $\mathcal{Y} \cap \alpha'\mathcal{X} \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \mathcal{X} \;\delta'\; \mathcal{Y}$. Analogously $\mathcal{X} \cap \beta'\mathcal{Y} \neq 0^{\mathfrak{F}(A)} \Leftrightarrow \mathcal{X} \;\delta'\; \mathcal{Y}$. So $\mathcal{Y} \cap \alpha'\mathcal{X} \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \mathcal{X} \cap \beta'\mathcal{Y} \neq 0^{\mathfrak{F}(A)}$, that is $(A; B; \alpha'; \beta')$ is a funcoid. From the formula $\mathcal{Y} \cap \alpha'\mathcal{X} \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \mathcal{X} \;\delta'\; \mathcal{Y}$ it follows that

$$X \left[(A; B; \alpha'; \beta')\right]^* Y \Leftrightarrow \uparrow^B Y \cap \alpha' \uparrow^A X \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \uparrow^A X \;\delta'\; \uparrow^B Y \Leftrightarrow X \;\delta\; Y.$$

1. Let define the relation $\delta \in \mathscr{P}(\mathscr{P}A \times \mathscr{P}B)$ by the formula $X \;\delta\; Y \Leftrightarrow \uparrow^B Y \cap \alpha X \neq 0^{\mathfrak{F}(B)}$.

That $\neg(\emptyset \;\delta\; I)$ and $\neg(I \;\delta\; \emptyset)$ is obvious. We have $I \cup J \;\delta\; K \Leftrightarrow \uparrow^B K \cap \alpha(I \cup J) \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \uparrow^B K \cap (\alpha I \cup \alpha J) \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \uparrow^B K \cap \alpha I \neq 0^{\mathfrak{F}(B)} \vee \uparrow^B K \cap \alpha I \neq 0^{\mathfrak{F}(B)} \Leftrightarrow I \;\delta\; K \vee J \;\delta\; K$ and

$K \;\delta\; I \cup J \Leftrightarrow \uparrow^B (I \cup J) \cap \alpha K \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \left(\uparrow^B I \cup \uparrow^B J\right) \cap \alpha K \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \left(\uparrow^B I \cap \alpha K\right) \cup \left(\uparrow^B J \cap \alpha K\right) \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \uparrow^B I \cap \alpha K \neq 0^{\mathfrak{F}(B)} \vee \uparrow^B J \cap \alpha K \neq 0^{\mathfrak{F}(B)} \Leftrightarrow K \;\delta\; I \vee K \;\delta\; J$.