Proposition 5 *$\langle f\rangle 0^{\mathfrak{F}(\mathrm{Src}\, f)} = 0^{\mathfrak{F}(\mathrm{Dst}\, f)}$ for every funcoid f.*

Proof $\mathcal{Y} \not\asymp \langle f\rangle 0^{\mathfrak{F}(\mathrm{Src}\, f)} \Leftrightarrow 0^{\mathfrak{F}(\mathrm{Src}\, f)} \not\asymp \langle f^{-1}\rangle \mathcal{Y} \Leftrightarrow 0 \Leftrightarrow \mathcal{Y} \not\asymp 0^{\mathfrak{F}(\mathrm{Dst}\, f)}$. Thus $\langle f\rangle 0^{\mathfrak{F}(\mathrm{Src}\, f)} = 0^{\mathfrak{F}(\mathrm{Dst}\, f)}$ by separability of filter objects. □

Proposition 6 *$\langle f\rangle (\mathcal{I} \cup \mathcal{J}) = \langle f\rangle \mathcal{I} \cup \langle f\rangle \mathcal{J}$ for every funcoid f and $\mathcal{I}, \mathcal{J} \in \mathfrak{F}(\mathrm{Src}\, f)$.*

Proof

$$\begin{aligned}
\star \langle f\rangle (\mathcal{I} \cup \mathcal{J}) &= \\
\{\mathcal{Y} \in \mathfrak{F} \mid \mathcal{Y} \not\asymp \langle f\rangle (\mathcal{I} \cup \mathcal{J})\} &= \\
\{\mathcal{Y} \in \mathfrak{F} \mid \mathcal{I} \cup \mathcal{J} \not\asymp \langle f^{-1}\rangle \mathcal{Y}\} &= \quad \text{(by corollary 10 in [15])} \\
\{\mathcal{Y} \in \mathfrak{F} \mid \mathcal{I} \not\asymp \langle f^{-1}\rangle \mathcal{Y} \vee \mathcal{J} \not\asymp \langle f^{-1}\rangle \mathcal{Y}\} &= \\
\{\mathcal{Y} \in \mathfrak{F} \mid \mathcal{Y} \not\asymp \langle f\rangle \mathcal{I} \vee \mathcal{Y} \not\asymp \langle f\rangle \mathcal{J}\} &= \\
\{\mathcal{Y} \in \mathfrak{F} \mid \mathcal{Y} \not\asymp \langle f\rangle \mathcal{I} \cup \langle f\rangle \mathcal{J}\} &= \\
\star(\langle f\rangle \mathcal{I} \cup \langle f\rangle \mathcal{J}).
\end{aligned}$$

Thus $\langle f\rangle (\mathcal{I} \cup \mathcal{J}) = \langle f\rangle \mathcal{I} \cup \langle f\rangle \mathcal{J}$ because $\mathfrak{F}(\mathrm{Dst}\, f)$ is separable. □

Proposition 7 *For every $f \in \mathsf{FCD}(A; B)$ for every small sets A and B we have:*

1. *$\mathcal{K} [f] \mathcal{I} \cup \mathcal{J} \Leftrightarrow \mathcal{K} [f] \mathcal{I} \vee \mathcal{K} [f] \mathcal{J}$ for every $\mathcal{I}, \mathcal{J} \in \mathfrak{F}(B)$, $\mathcal{K} \in \mathfrak{F}(A)$.*
2. *$\mathcal{I} \cup \mathcal{J} [f] \mathcal{K} \Leftrightarrow \mathcal{I} [f] \mathcal{K} \vee \mathcal{J} [f] \mathcal{K}$ for every $\mathcal{I}, \mathcal{J} \in \mathfrak{F}(A)$, $\mathcal{K} \in \mathfrak{F}(B)$.*

Proof 1. $\mathcal{K} [f] \mathcal{I} \cup \mathcal{J} \Leftrightarrow (\mathcal{I} \cup \mathcal{J}) \cap \langle f\rangle \mathcal{K} \neq 0^{\mathfrak{F}(B)} \Leftrightarrow (\mathcal{I} \cap \langle f\rangle \mathcal{K}) \cup (\mathcal{J} \cap \langle f\rangle \mathcal{K}) \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \mathcal{I} \cap \langle f\rangle \mathcal{K} \neq 0^{\mathfrak{F}(B)} \vee \mathcal{J} \cap \langle f\rangle \mathcal{K} \neq 0^{\mathfrak{F}(B)} \Leftrightarrow \mathcal{K} [f] \mathcal{I} \vee \mathcal{K} [f] \mathcal{J}$.

2. Similar. □

3.2.1 Composition of funcoids

Definition 20 *Funcoids f and g are **composable** when $\mathrm{Dst}\, f = \mathrm{Src}\, g$.*

Definition 21 ***Composition** of composable funcoids is defined by the formula*

$$(B; C; \alpha_2; \beta_2) \circ (A; B; \alpha_1; \beta_1) = (A; C; \alpha_2 \circ \alpha_1; \beta_1 \circ \beta_2).$$

Proposition 8 *If f, g are composable funcoids then $g \circ f$ is a funcoid.*

Proof Let $f = (A; B; \alpha_1; \beta_1)$, $g = (B; C; \alpha_2; \beta_2)$. For every $\mathcal{X} \in \mathfrak{F}(A)$, $\mathcal{Y} \in \mathfrak{F}(C)$ we have

$$\mathcal{Y} \not\asymp (\alpha_2 \circ \alpha_1)\mathcal{X} \Leftrightarrow \mathcal{Y} \not\asymp \alpha_2\alpha_1\mathcal{X} \Leftrightarrow \alpha_1\mathcal{X} \not\asymp \beta_2\mathcal{Y} \Leftrightarrow \mathcal{X} \not\asymp \beta_1\beta_2\mathcal{Y} \Leftrightarrow \mathcal{X} \not\asymp (\beta_1 \circ \beta_2)\mathcal{Y}.$$