3.2 Basic definitions

Definition 15 *Let's call **a funcoid** from a set A to a set B a quadruple $(A; B; \alpha; \beta)$ where $\alpha \in \mathfrak{F}(B)^{\mathfrak{F}(A)}$, $\beta \in \mathfrak{F}(A)^{\mathfrak{F}(B)}$ such that*

$$\forall \mathcal{X} \in \mathfrak{F}(A), \mathcal{Y} \in \mathfrak{F}(B) : (\mathcal{Y} \not\asymp \alpha\mathcal{X} \Leftrightarrow \mathcal{X} \not\asymp \beta\mathcal{Y}).$$

Further we will assume that all funcoids in consideration are small without mentioning it explicitly.

Definition 16 ***Source*** *and* ***destination*** *of every funcoid $(A; B; \alpha; \beta)$ are defined as*

$$\operatorname{Src}(A; B; \alpha; \beta) = A \quad \text{and} \quad \operatorname{Dst}(A; B; \alpha; \beta) = B.$$

I will denote $\mathsf{FCD}(A; B)$ the set of funcoids from A to B.

I will denote FCD the set of all funcoids (for small sets).

Definition 17 $\langle (A; B; \alpha; \beta) \rangle \stackrel{\text{def}}{=} \alpha$ *for a funcoid $(A; B; \alpha; \beta)$.*

Definition 18 $(A; B; \alpha; \beta)^{-1} = (B; A; \beta; \alpha)$ *for a funcoid $(A; B; \alpha; \beta)$.*

Proposition 4 *If f is a funcoid then f^{-1} is also a funcoid.*

Proof It follows from symmetry in the definition of funcoid. □

Obvious 4. $(f^{-1})^{-1} = f$ for a funcoid f.

Definition 19 *The relation $[f] \in \mathscr{P}(\mathfrak{F}(\operatorname{Src} f) \times \mathfrak{F}(\operatorname{Dst} f))$ is defined (for every funcoid f and $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$, $\mathcal{Y} \in \mathfrak{F}(\operatorname{Dst} f)$) by the formula $\mathcal{X}\ [f]\ \mathcal{Y} \stackrel{\text{def}}{=} \mathcal{Y} \not\asymp \langle f \rangle \mathcal{X}$.*

Obvious 5. $\mathcal{X}\ [f]\ \mathcal{Y} \Leftrightarrow \mathcal{Y} \not\asymp \langle f \rangle \mathcal{X} \Leftrightarrow \mathcal{X} \not\asymp \langle f^{-1} \rangle \mathcal{Y}$ for every funcoid f and $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$, $\mathcal{Y} \in \mathfrak{F}(\operatorname{Dst} f)$.

Obvious 6. $[f^{-1}] = [f]^{-1}$ for a funcoid f.

Theorem 4 *Let A, B are small sets.*

1. *For given value of $\langle f \rangle$ exists no more than one funcoid $f \in \mathsf{FCD}(A; B)$.*
2. *For given value of $[f]$ exists no more than one funcoid $f \in \mathsf{FCD}(A; B)$.*

Proof Let $f, g \in \mathsf{FCD}(A; B)$.

Obviously $\langle f \rangle = \langle g \rangle \Rightarrow [f] = [g]$ and $\langle f^{-1} \rangle = \langle g^{-1} \rangle \Rightarrow [f] = [g]$. So it's enough to prove that $[f] = [g] \Rightarrow \langle f \rangle = \langle g \rangle$.

Provided that $[f] = [g]$ we have $\mathcal{Y} \not\asymp \langle f \rangle \mathcal{X} \Leftrightarrow \mathcal{X}\ [f]\ \mathcal{Y} \Leftrightarrow \mathcal{X}\ [g]\ \mathcal{Y} \Leftrightarrow \mathcal{Y} \not\asymp \langle g \rangle \mathcal{X}$ and consequently $\langle f \rangle \mathcal{X} = \langle g \rangle \mathcal{X}$ for every $\mathcal{X} \in \mathfrak{F}(A)$ and $\mathcal{Y} \in \mathfrak{F}(B)$ because a set of filter objects is separable [15], thus $\langle f \rangle = \langle g \rangle$. □