Let δ be a proximity that is certain binary relation so that $A\ \delta\ B$ is defined for every sets A and B. We will extend it from sets to filter objects by the formula:

$$\mathcal{A}\ \delta'\ \mathcal{B} \Leftrightarrow \forall A \in \operatorname{up}\mathcal{A}, B \in \operatorname{up}\mathcal{B} : A\ \delta\ B.$$

Then (as it will be proved below) there exist two functions $\alpha, \beta \in \mathfrak{F}^{\mathfrak{F}}$ such that

$$\mathcal{A}\ \delta'\ \mathcal{B} \Leftrightarrow \mathcal{B} \cap^{\mathfrak{F}} \alpha\mathcal{A} \neq 0^{\mathfrak{F}} \Leftrightarrow \mathcal{A} \cap^{\mathfrak{F}} \beta\mathcal{B} \neq 0^{\mathfrak{F}}.$$

The pair $(\alpha; \beta)$ is called **funcoid** when $\mathcal{B} \cap^{\mathfrak{F}} \alpha\mathcal{A} \neq 0^{\mathfrak{F}} \Leftrightarrow \mathcal{A} \cap^{\mathfrak{F}} \beta\mathcal{B} \neq 0^{\mathfrak{F}}$. So funcoids are a generalization of proximity spaces.

Funcoids consist of two components the first α and the second β. The first component of a funcoid f is denoted as $\langle f \rangle$ and the second component is denoted as $\langle f^{-1} \rangle$. (The similarity of this notation with the notation for the image of a set under a function is not a coincidence, we will see that in the case of principal funcoids (see below) these coincide.)

One of the most important properties of a funcoid is that it is uniquely determined by just one of its components. That is a funcoid f is uniquely determined by the function $\langle f \rangle$. Moreover a funcoid f is uniquely determined by $\langle f \rangle|_{\mathscr{P}\bigcup \operatorname{dom}\langle f \rangle}$ that is by values of function $\langle f \rangle$ on sets (if we equate principal filters with sets).

Next we will consider some examples of funcoids determined by specified values of the first component on sets.

Funcoids as a generalization of pretopological spaces: Let α be a pretopological space that is a map $\alpha \in \mathfrak{F}^{\mho}$ for some set $\mho$. Then we define $\alpha' X \stackrel{\text{def}}{=} \bigcup^{\mathfrak{F}} \{ \alpha x \mid x \in X \}$ for every set $X \in \mathscr{P}\mho$. We will prove that there exists a unique funcoid f such that $\alpha' = \langle f \rangle|_{\mathscr{P}\mho}$. So funcoids are a generalization of pretopological spaces. Funcoids are also a generalization of preclosure operators: For every preclosure operator p on a set $\mho$ it exists a unique funcoid f such that $\langle f \rangle|_{\mathscr{P}\mho} = \uparrow \circ p$.

For every binary relation p on a set $\mho$ it exists unique funcoid f such that $\forall X \in \mathscr{P}\mho : \langle f \rangle \uparrow X = \uparrow \langle p \rangle X$ (where $\langle p \rangle$ is defined in the introduction), recall that a funcoid is uniquely determined by the values of its first component on sets. I will call such funcoids **principal**. So funcoids are a generalization of binary relations.

Composition of binary relations (i.e. of principal funcoids) complies with the formulas:

$$\langle g \circ f \rangle = \langle g \rangle \circ \langle f \rangle \quad \text{and} \quad \left\langle (g \circ f)^{-1} \right\rangle = \left\langle f^{-1} \right\rangle \circ \left\langle g^{-1} \right\rangle.$$

By the same formulas we can define composition of every two funcoids. Funcoids with this composition form a category (**the category of funcoids**).

Also funcoids can be reversed (like reversal of X and Y in a binary relation) by the formula $(\alpha; \beta)^{-1} = (\beta; \alpha)$. In particular case if μ is a proximity we have $\mu^{-1} = \mu$ because proximities are symmetric.

Funcoids behave similarly to (multivalued) functions but acting on filter objects instead of acting on sets. Below these will be defined domain and image of a funcoid (the domain and the image of a funcoid are filter objects).