We need to prove that these are really categories, that is that composition of monovalued (entirely defined, injective, surjective) morphisms is monovalued (entirely defined, injective, surjective) and that identity morphisms are monovalued, entirely defined, injective, and surjective.

Proof We will prove only for monovalued morphisms and entirely defined morphisms, as injective and surjective morphisms are their duals.

Monovalued Let f and g are monovalued morphisms, $\operatorname{Dst} f = \operatorname{Src} g$. $(g \circ f) \circ (g \circ f)^\dagger = g \circ f \circ f^\dagger \circ g^\dagger \subseteq g \circ 1_{\operatorname{Dst} f} \circ g^\dagger = g \circ 1_{\operatorname{Src} g} \circ g^\dagger = g \circ g^\dagger \subseteq 1_{\operatorname{Dst} g} = 1_{\operatorname{Dst}(g \circ f)}$. So $g \circ f$ is monovalued.

That identity morphisms are monovalued follows from the following: $1_A \circ (1_A)^\dagger = 1_A \circ 1_A = 1_A = 1_{\operatorname{Dst} 1_A} \subseteq 1_{\operatorname{Dst} 1_A}$.

Entirely defined Let f and g are entirely defined morphisms, $\operatorname{Dst} f = \operatorname{Src} g$. $(g \circ f)^\dagger \circ (g \circ f) = f^\dagger \circ g^\dagger \circ g \circ f \supseteq f^\dagger \circ 1_{\operatorname{Src} g} \circ f = f^\dagger \circ 1_{\operatorname{Dst} f} \circ f = f^\dagger \circ f \supseteq 1_{\operatorname{Src} f} = 1_{\operatorname{Src}(g \circ f)}$. So $g \circ f$ is entirely defined.

That identity morphisms are entirely defined follows from the following: $(1_A)^\dagger \circ 1_A = 1_A \circ 1_A = 1_A = 1_{\operatorname{Src} 1_A} \supseteq 1_{\operatorname{Src} 1_A}$.

□

Definition 14 *I will call a* ***bijective*** *morphism a morphism which is entirely defined, monovalued, injective, and surjective.*

Obvious 3. Bijective morphisms form a full subcategory.

Proposition 3 *If a morphism is bijective then it is an isomorphism.*

Proof Let f is bijective. Then $f \circ f^\dagger \subseteq 1_{\operatorname{Dst} f}$, $f^\dagger \circ f \supseteq 1_{\operatorname{Src} f}$, $f^\dagger \circ f \subseteq 1_{\operatorname{Src} f}$, $f \circ f^\dagger \supseteq 1_{\operatorname{Dst} f}$. Thus $f \circ f^\dagger = 1_{\operatorname{Dst} f}$ and $f^\dagger \circ f = 1_{\operatorname{Src} f}$ that is $f^\dagger$ is an inverse of f. □

3 Funcoids

3.1 Informal introduction into funcoids

Funcoids are a generalization of proximity spaces and a generalization of pretopological spaces. Also funcoids are a generalization of binary relations.

That funcoids are a common generalization of "spaces" (proximity spaces, (pre)topological spaces) and binary relations (including monovalued functions) makes them smart for describing properties of functions in regard of spaces. For example the statement "f is a continuous function from a space μ to a space ν" can be described in terms of funcoids as the formula $f \circ \mu \subseteq \nu \circ f$ (see below for details).

Most naturally funcoids appear as a generalization of proximity spaces.