Definition 8 ***Transitive*** *(endo)morphism of a precategory is such a morphism* f *that* $f = f \circ f$.

Theorem 3 *The following conditions are equivalent for a morphism* f *of a dagger precategory:*

1. f *is symmetric and transitive.*

2. $f = f^\dagger \circ f$.

Proof

(1)⇒(2) If f is symmetric and transitive then $f^\dagger \circ f = f \circ f = f$.

(2)⇒(1) $f^\dagger = (f^\dagger \circ f)^\dagger = f^\dagger \circ f^{\dagger\dagger} = f^\dagger \circ f = f$, so f is symmetric. $f = f^\dagger \circ f = f \circ f$, so f is transitive.

□

2.2.1 Some special classes of morphisms

Definition 9 *For a partially ordered dagger category I will call* ***monovalued*** *morphism such a morphism* f *that* $f \circ f^\dagger \subseteq 1_{\mathrm{Dst}\, f}$.

Definition 10 *For a partially ordered dagger category I will call* ***entirely defined*** *morphism such a morphism* f *that* $f^\dagger \circ f \supseteq 1_{\mathrm{Src}\, f}$.

Definition 11 *For a partially ordered dagger category I will call* ***injective*** *morphism such a morphism* f *that* $f^\dagger \circ f \subseteq 1_{\mathrm{Src}\, f}$.

Definition 12 *For a partially ordered dagger category I will call* ***surjective*** *morphism such a morphism* f *that* $f \circ f^\dagger \supseteq 1_{\mathrm{Dst}\, f}$.

Remark 1 It's easy to show that this is a generalization of monovalued, entirely defined, injective, and surjective binary relations as morphisms of the category **Rel**.

Obvious 2. "Injective morphism" is a dual of "monovalued morphism" and "surjective morphism" is a dual of "entirely defined morphism".

Definition 13 *For a given partially ordered dagger category* C *the* ***category of monovalued (entirely defined, injective, surjective) morphisms*** *of* C *is the category with the same set of objects as of* C *and the set of morphisms being the set of monovalued (entirely defined, injective, surjective) morphisms of* C *with the composition of morphisms the same as in* C.