Also we will need to introduce the concept of *generalized filter base*.

Definition 1 ***Generalized filter base*** *is a set* $S \in \mathscr{P}\mathfrak{F} \setminus \left\{ 0^{\mathfrak{F}} \right\}$ *such that*

$$\forall \mathcal{A}, \mathcal{B} \in S \exists \mathcal{C} \in S : \mathcal{C} \subseteq \mathcal{A} \cap \mathcal{B}.$$

Proposition 2 *Let* S *is a generalized filter base. If* $\mathcal{A}_1, \ldots, \mathcal{A}_n \in S$ $(n \in \mathbb{N})$*, then*

$$\exists \mathcal{C} \in S : \mathcal{C} \subseteq \mathcal{A}_1 \cap \ldots \cap \mathcal{A}_n.$$

Proof Can be easily proved by induction. □

Theorem 1 *If* S *is a generalized filter base, then* $\operatorname{up} \bigcap S = \bigcup \langle \operatorname{up} \rangle S$*.*

Proof Obviously $\operatorname{up} \bigcap S \supseteq \bigcup \langle \operatorname{up} \rangle S$. Reversely, let $K \in \operatorname{up} \bigcap S$; then $K = A_1 \cap \ldots \cap A_n$ where $A_i \in \operatorname{up} \mathcal{A}_i$ where $\mathcal{A}_i \in S$, $i = 1, \ldots, n$, $n \in \mathbb{N}$; so exists $\mathcal{C} \in S$ such that $\mathcal{C} \subseteq \mathcal{A}_1 \cap \ldots \cap \mathcal{A}_n \subseteq \uparrow (A_1 \cap \ldots \cap A_n) = \uparrow K$, $K \in \operatorname{up} \mathcal{C}$, $K \in \bigcup \langle \operatorname{up} \rangle S$. □

Corollary 1 *If* S *is a generalized filter base, then* $\bigcap S = 0^{\mathfrak{F}} \Leftrightarrow 0^{\mathfrak{F}} \in S$*.*

Proof $\bigcap S = 0^{\mathfrak{F}} \Leftrightarrow \emptyset \in \operatorname{up} \bigcap S \Leftrightarrow \emptyset \in \bigcup \langle \operatorname{up} \rangle S \Leftrightarrow \exists \mathcal{X} \in S : \emptyset \in \operatorname{up} \mathcal{X} \Leftrightarrow 0^{\mathfrak{F}} \in S$. □

Obvious 1. If S is a filter base on a set A then $\left\langle \uparrow^A \right\rangle S$ is a generalized filter base.

Definition 2 *I will call* ***shifted filtrator*** *a triple* $(\mathfrak{A}; \mathfrak{Z}; \uparrow)$ *where* $\mathfrak{A}$ *and* $\mathfrak{Z}$ *are posets and* $\uparrow$ *is an order embedding from* $\mathfrak{Z}$ *to* $\mathfrak{A}$*.*

Some concepts and notation can be defined for shifted filtrators through similar concepts for filtrators: $\langle \uparrow \rangle \operatorname{up} a = \operatorname{up}^{(\mathfrak{A}; \langle \uparrow \rangle \mathfrak{Z})} a$; $\langle \uparrow \rangle \operatorname{Cor} a = \operatorname{Cor}^{(\mathfrak{A}; \langle \uparrow \rangle \mathfrak{Z})} a$, etc.

For a set $\mathfrak{A}$ and the set of f.o. $\mathfrak{F}$ on this set we will consider the shifted filtrator $(\mathfrak{F}; \mathfrak{A}; \uparrow)$.

2 Partially ordered dagger categories

2.1 Partially ordered categories

Definition 3 *I will call a* ***partially ordered (pre)category*** *a (pre)category together with partial order* $\subseteq$ *on each of its Hom-sets with the additional requirement that*

$$f_1 \subseteq f_2 \wedge g_1 \subseteq g_2 \Rightarrow g_1 \circ f_1 \subseteq g_2 \circ f_2$$

for every morphisms f_1*,* g_1*,* f_2*,* g_2 *such that* $\operatorname{Src} f_1 = \operatorname{Src} f_2 \wedge \operatorname{Dst} f_1 = \operatorname{Dst} f_2 = \operatorname{Src} g_1 = \operatorname{Src} g_2 \wedge \operatorname{Dst} g_1 = \operatorname{Dst} g_2$*.*