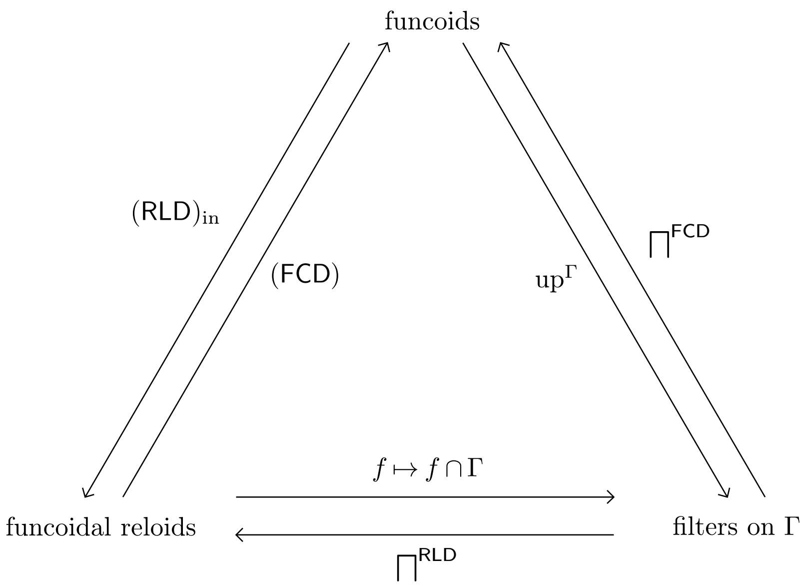

Proof. First we need to show that $\prod^{\mathsf{RLD}} f$ is a funcoidal reloid. But it follows from lemma 25.

Next, we need to show that all morphisms depicted on the diagram are bijections and the depicted "opposite" morphisms are mutually inverse.

That (FCD) and $(\mathsf{RLD})_{\mathrm{in}}$ are mutually inverse was proved above in the book.

That $\prod^{\mathsf{RLD}}$ and $f \mapsto f \cap \Gamma$ are mutually inverse was proved above.

That $\prod^{\mathsf{FCD}}$ and up^{Γ} are mutually inverse was proved above.

It remains to prove that three-element cycles are identities. But this follows from lemma 25.

That the morphisms preserve order and composition was proved above. That they preserve reversal is obvious. □

6 Some additional properties

Proposition 36. For every funcoid $f \in \mathsf{FCD}(A; B)$ (for sets A, B):

1. $\mathrm{dom}\, f = \prod^{\mathfrak{F}(A)} \langle \mathrm{dom} \rangle^* \mathrm{up}^{\Gamma(A;B)} f$;
2. $\mathrm{im}\, f = \prod^{\mathfrak{F}(A)} \langle \mathrm{im} \rangle^* \mathrm{up}^{\Gamma(A;B)} f$.

Proof. Take $\{X \times Y \mid X \in \mathscr{P}A, Y \in \mathscr{P}A, X \times Y \sqsupseteq f\} \subseteq \mathrm{up}^{\Gamma(A;B)} f$. I leave the rest reasoning as an exercise. □

Proposition 37. $(\mathsf{RLD})_{\Gamma} f \sqsupseteq (\mathsf{RLD})_{\mathrm{in}} f \sqsupseteq (\mathsf{RLD})_{\mathrm{out}} f$ for every funcoid f.

Proof. We already know that $(\mathsf{RLD})_{\mathrm{in}} f \sqsupseteq (\mathsf{RLD})_{\mathrm{out}} f$ (see above in the book).

The formula $(\mathsf{RLD})_{\Gamma} f \sqsupseteq (\mathsf{RLD})_{\mathrm{in}} f$ follows from $\forall G \in \mathrm{up}^{\Gamma(\mathrm{Src}\, f; \mathrm{Dst}\, f)} f \colon G \sqsupseteq f$. □

Example 38. $(\mathsf{RLD})_{\Gamma} f \sqsupset (\mathsf{RLD})_{\mathrm{in}} f \sqsupset (\mathsf{RLD})_{\mathrm{out}} f$ for some funcoid f.

Proof. Take $f = (=)|_{\mathbb{R}}$. We already know that $(\mathsf{RLD})_{\mathrm{in}} f \sqsupset (\mathsf{RLD})_{\mathrm{out}} f$ (see above in the book).

It remains to prove $(\mathsf{RLD})_{\Gamma} f \neq (\mathsf{RLD})_{\mathrm{in}} f$.

Take $F = \bigcup_{i \in \mathbb{Z}} ([i; i+1[\times [i; i+1[)$.

Then $F \in f = \mathrm{up}\, (\mathsf{RLD})_{\mathrm{in}} f$ (because $\langle F \rangle a \sqsupseteq \langle f \rangle a$ for both principal ultrafilter $a = \{i\}$ and every other ultrafilter a).

It remains to prove $F \notin \mathrm{up}\, (\mathsf{RLD})_{\Gamma} f$.