So by properties of generalized filter bases, there exists $A \in \operatorname{up} a$ such that $Y \in \operatorname{up} \langle f \rangle A$. □

Lemma 31. $(\mathsf{FCD}) f = \bigsqcap^{\mathsf{FCD}} (\Gamma(A; B) \cap \operatorname{GR} f)$ for every reloid $f \in \mathsf{RLD}(A; B)$.

Proof. Let a be an ultrafilter. We need to prove

$$\langle (\mathsf{FCD}) f \rangle a = \left\langle \bigsqcap^{\mathsf{FCD}} (\Gamma(A; B) \cap \operatorname{GR} f) \right\rangle a$$

that is

$$\left\langle \bigsqcap^{\mathsf{FCD}} \operatorname{up} f \right\rangle a = \left\langle \bigsqcap^{\mathsf{FCD}} (\Gamma(A; B) \cap \operatorname{GR} f) \right\rangle a$$

that is

$$\bigsqcap_{F \in \operatorname{up} f}^{\mathfrak{F}} \langle F \rangle a = \bigsqcap_{F \in \Gamma(A; B) \cap \operatorname{up} f}^{\mathfrak{F}} \langle F \rangle a.$$

For this it's enough to prove that $Y \in \operatorname{up} \langle F \rangle a$ for some $F \in \operatorname{up} f$ implies $Y \in \operatorname{up} \langle F' \rangle a$ for some $F' \in \Gamma(A; B) \cap \operatorname{GR} f$.

Let $Y \in \operatorname{up} \langle F \rangle a$. Then (proposition above) there exists $A \in \operatorname{up} a$ such that $Y \in \operatorname{up} \langle F \rangle A$.

$Y \in \operatorname{up} \langle A \times^{\mathsf{FCD}} Y \sqcup \overline{A} \times^{\mathsf{FCD}} 1 \rangle a$; $\langle A \times^{\mathsf{FCD}} Y \sqcup \overline{A} \times^{\mathsf{FCD}} 1 \rangle \mathcal{X} = Y \in \operatorname{up} \langle F \rangle \mathcal{X}$ if $0 \neq \mathcal{X} \sqsubseteq A$ and $\langle A \times^{\mathsf{FCD}} Y \sqcup \overline{A} \times^{\mathsf{FCD}} 1 \rangle \mathcal{X} = 1 \in \operatorname{up} \langle F \rangle \mathcal{X}$ if $\mathcal{X} \not\sqsubseteq A$.

Thus $A \times^{\mathsf{FCD}} Y \sqcup \overline{A} \times^{\mathsf{FCD}} 1 \sqsupseteq F$. So $A \times^{\mathsf{FCD}} Y \sqcup \overline{A} \times^{\mathsf{FCD}} 1$ is the sought for F'. □

4.2 Relationships between (FCD) and $(\mathsf{RLD})_\Gamma$

Definition 32. $(\mathsf{RLD})_\Gamma f = \bigsqcap^{\mathsf{RLD}} \operatorname{up}^{\Gamma(\operatorname{Src} f; \operatorname{Dst} f)} f$ for every funcoid f. I call $(\mathsf{RLD})_\Gamma$ as Γ-reloid or Gamma-reloid.

Lemma 33. $(\mathsf{FCD})(\mathsf{RLD})_\Gamma f = f$ for every funcoid f.

Proof. For every filter $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$ we have $\langle (\mathsf{FCD})(\mathsf{RLD})_\Gamma f \rangle \mathcal{X} = \bigsqcap^{\mathfrak{F}}_{F \in \operatorname{up} (\mathsf{RLD})_\Gamma f} \langle F \rangle \mathcal{X} = \bigsqcap^{\mathfrak{F}}_{F \in \operatorname{up}^{\Gamma(\operatorname{Src} f; \operatorname{Dst} f)} f} \langle F \rangle \mathcal{X}$.

Obviously $\bigsqcap^{\mathfrak{F}}_{F \in \operatorname{up}^{\Gamma(\operatorname{Src} f; \operatorname{Dst} f)} f} \langle F \rangle \mathcal{X} \sqsupseteq \langle f \rangle \mathcal{X}$. So $(\mathsf{FCD})(\mathsf{RLD})_\Gamma f \sqsupseteq f$.

Let $Y \in \operatorname{up} \langle f \rangle \mathcal{X}$. Then (propositiona above) there exists $A \in \operatorname{up} \mathcal{X}$ such that $Y \in \operatorname{up} \langle f \rangle A$.

Thus $A \times Y \cup \overline{A} \times 1 \in \operatorname{up} f$. So $\langle (\mathsf{FCD})(\mathsf{RLD})_\Gamma f \rangle \mathcal{X} = \bigsqcap^{\mathfrak{F}}_{F \in \operatorname{up}^{\Gamma(\operatorname{Src} f; \operatorname{Dst} f)} f} \langle F \rangle \mathcal{X} \sqsubseteq \langle A \times Y \cup \overline{A} \times 1 \rangle \mathcal{X} = Y$. So $Y \in \operatorname{up} \langle (\mathsf{FCD})(\mathsf{RLD})_\Gamma f \rangle \mathcal{X}$ that is $\langle f \rangle \mathcal{X} \sqsupseteq \langle (\mathsf{FCD})(\mathsf{RLD})_\Gamma f \rangle \mathcal{X}$ that is $f \sqsupseteq (\mathsf{FCD})(\mathsf{RLD})_\Gamma f$. □

Proposition 34. $(\mathsf{RLD})_\Gamma$ is neither upper nor lower adjoint of (FCD) (in general).

Proof. It is not upper adjoint because $(\mathsf{RLD})_{\mathrm{in}}$ is the upper adjoint of (FCD) and $(\mathsf{RLD})_{\mathrm{in}} \neq (\mathsf{RLD})_\Gamma$.

If $(\mathsf{RLD})_\Gamma$ is the lower adjoint of (FCD), then $f \sqsupseteq (\mathsf{RLD})_\Gamma (\mathsf{FCD}) f$ and thus $f \sqsupseteq (\mathsf{RLD})_{\mathrm{in}} (\mathsf{FCD}) f$. But $f \sqsubseteq (\mathsf{RLD})_{\mathrm{in}} (\mathsf{FCD}) f$, thus having $(\mathsf{RLD})_{\mathrm{in}} (\mathsf{FCD}) f = f$ what is not an identity (take $f = (=)|_A$ for an infinite set A). □

5 The diagram

Theorem 35. The following is a commutative diagram (in category **Set**), every arrow in this diagram is an isomorphism. Every cycle in this diagram is an identity (therefore "parallel" arrows are mutually inverse). The arrows preserve order, composition, and reversal ($f \mapsto f^{-1}$).