I will denote $\mathfrak{F}\Gamma(A;B) = \{(A;B;F) \mid F \in \mathfrak{F}\Gamma[A;B]\}$.

Remark 12. It should be instead be denoted as $(\mathfrak{F}\circ\Gamma)(A;B)$ but for brevity I omit $\circ$.

4 Before the diagram

Next we will prove the below theorem 35 (the theorem with a diagram). First we will present parts of this theorem as several lemmas, and then then state a statement about the diagram which concisely summarizes the lemmas (and their easy consequences).

Obvious 13. $\operatorname{up}^{\Gamma(\operatorname{Src} f;\operatorname{Dst} f)} f = (\operatorname{up} f) \cap \Gamma$ for every reloid f.

Conjecture 14. $\upuparrows^{\mathfrak{F}(\mathfrak{B})} \operatorname{up}^{\mathfrak{A}} \mathcal{X}$ is not a filter for some filter $\mathcal{X} \in \mathfrak{F}\Gamma(A;B)$ for some sets A, B.

Remark 15. About this conjecture see also:

- http://goo.gl/DHyuuU
- http://goo.gl/4a6wY6

Lemma 16. Let A, B be sets. The following are mutually inverse order isomorphisms between $\mathfrak{F}\Gamma(A;B)$ and $\mathsf{FCD}(A;B)$:

1. $\mathcal{A} \mapsto \bigsqcap^{\mathsf{FCD}} \operatorname{up} \mathcal{A}$;
2. $f \mapsto \operatorname{up}^{\Gamma(A;B)} f$.

Proof. Let's prove that $\operatorname{up}^{\Gamma(A;B)} f$ is a filter for every funcoid f. We need to prove that $P \cap Q \in \operatorname{up} f$ whenever

$$P = \bigcap_{i=0,\ldots,n-1} (X_i \times Y_i \cup \overline{X_i} \times B) \quad \text{and} \quad Q = \bigcap_{j=0,\ldots,m-1} (X'_j \times Y'_j \cup \overline{X'_j} \times B).$$

This follows from $P \in \operatorname{up} f \Leftrightarrow \forall i \in 0, \ldots, n-1 \colon \langle f\rangle X_i \subseteq Y_i$ and likewise for Q, so having $\langle f\rangle (X_i \cap X'_j) \subseteq Y_i \cap Y'_j$ for every $i = 0, \ldots, n-1$ and $j = 0, \ldots, m-1$. From this it follows

$$((X_i \cap X'_j) \times (Y_i \cap Y'_j)) \cup (\overline{X_i \cap X'_j} \times B) \supseteq f$$

and thus $P \cap Q \in \operatorname{up} f$.

Let $\mathcal{A}$, $\mathcal{B}$ be filters on Γ. Let $\bigsqcap^{\mathsf{FCD}} \operatorname{up} \mathcal{A} = \bigsqcap^{\mathsf{FCD}} \operatorname{up} \mathcal{B}$. We need to prove $\mathcal{A} = \mathcal{B}$. (The rest follows from proof of the theorem 6.104 from my book [1]). We have: [TODO: Separate the first equality below from theorem 6.104 into a separate lemma.]

$$\mathcal{A} = \bigsqcap^{\mathsf{FCD}} \{X \times Y \cup \overline{X} \times B \in \mathcal{A} \mid X \in \mathscr{P}A, Y \in \mathscr{P}B\} =$$

$$\bigsqcap^{\mathsf{FCD}} \{X \times Y \cup \overline{X} \times B \mid X \in \mathscr{P}A, Y \in \mathscr{P}B, \exists P \in \mathcal{A} \colon P \subseteq X \times Y \cup \overline{X} \times B\} =$$

$$\bigsqcap^{\mathsf{FCD}} \{X \times Y \cup \overline{X} \times B \mid X \in \mathscr{P}A, Y \in \mathscr{P}B, \exists P \in \mathcal{A} \colon \langle P\rangle^* X \subseteq Y\} = (*)$$

$$\bigsqcap^{\mathsf{FCD}} \{X \times Y \cup \overline{X} \times B \mid X \in \mathscr{P}A, Y \in \mathscr{P}B, \bigsqcap \{\langle P\rangle^* X \mid A \in \operatorname{up} \mathcal{A}\} \sqsubseteq Y\} =$$

$$\bigsqcap^{\mathsf{FCD}} \left\{X \times Y \cup \overline{X} \times B \mid X \in \mathscr{P}A, Y \in \mathscr{P}B, \bigsqcap \left\{\langle P\rangle^* X \mid A \in \operatorname{up} \bigsqcap^{\mathsf{RLD}} \operatorname{up} \mathcal{A}\right\} \sqsubseteq Y\right\} =$$

$$\bigsqcap^{\mathsf{FCD}} \left\{X \times Y \cup \overline{X} \times B \mid X \in \mathscr{P}A, Y \in \mathscr{P}B, \left\langle (\mathsf{FCD}) \bigsqcap^{\mathsf{RLD}} \operatorname{up} \mathcal{A} \right\rangle X \sqsubseteq Y\right\} = (**)$$

$$\bigsqcap^{\mathsf{FCD}} \left\{X \times Y \cup \overline{X} \times B \mid X \in \mathscr{P}A, Y \in \mathscr{P}B, \left\langle \bigsqcap^{\mathsf{FCD}} \operatorname{up} \bigsqcap^{\mathsf{RLD}} \operatorname{up} \mathcal{A} \right\rangle X \sqsubseteq Y\right\} =$$

$$\bigsqcap^{\mathsf{FCD}} \left\{X \times Y \cup \overline{X} \times B \mid X \in \mathscr{P}A, Y \in \mathscr{P}B, \left\langle \bigsqcap^{\mathsf{FCD}} \operatorname{up} \mathcal{A} \right\rangle X \sqsubseteq Y\right\}.$$