A generalization of theorem 43 in [1]:

Theorem 5. Let $(\mathfrak{A};\mathfrak{Z})$ be a starrish join-semilattice filtrator with finitely join-closed core which is a join-semilattice. Then ∂a is a free star for each $a\in\mathfrak{A}$.

Proof. For every $A,B\in\mathfrak{Z}$

$$\begin{aligned} X\cup^{\mathfrak{Z}}Y\in\partial a &\Leftrightarrow\\ X\cup^{\mathfrak{A}}Y\in\partial a &\Leftrightarrow\\ X\cup^{\mathfrak{A}}Y\not\asymp^{\mathfrak{A}}a &\Leftrightarrow\\ X\not\asymp^{\mathfrak{A}}a\vee Y\not\asymp^{\mathfrak{A}}a &\Leftrightarrow\\ X\in\partial a\vee Y\in\partial a. \end{aligned}$$

□

A generalization of theorem 65 in [1]:

Theorem 6. Let $(\mathfrak{A};\mathfrak{Z})$ be a semifiltered down-aligned filtrator with finitely meet-closed core $\mathfrak{Z}$ which is an atomistic lattice and $\mathfrak{A}$ is a starrish join-semilattice, then $\operatorname{Cor}'(a\cup^{\mathfrak{A}}b)=\operatorname{Cor}'a\cup^{\mathfrak{Z}}\operatorname{Cor}'b$ for every $a,b\in\mathfrak{A}$.

Proof. $\operatorname{Cor}'(a\cup^{\mathfrak{A}}b)=\bigcup^{\mathfrak{Z}}\{x\mid x\text{ is an atom of }\mathfrak{Z},x\subseteq a\cup^{\mathfrak{A}}b\}$ (use proposition 34 from [1]),

By the theorem 50 from [1] we have $\operatorname{Cor}'(a\cup^{\mathfrak{A}}b)=\bigcup^{\mathfrak{Z}}(\operatorname{atoms}^{\mathfrak{A}}(a\cup^{\mathfrak{A}}b)\cap\mathfrak{Z})=\bigcup^{\mathfrak{Z}}((\operatorname{atoms}^{\mathfrak{A}}a\cup\operatorname{atoms}b)\cap\mathfrak{Z})=\bigcup^{\mathfrak{Z}}((\operatorname{atoms}^{\mathfrak{A}}a\cap\mathfrak{Z})\cup(\operatorname{atoms}^{\mathfrak{A}}b\cap\mathfrak{Z}))=\bigcup^{\mathfrak{Z}}(\operatorname{atoms}^{\mathfrak{A}}a\cap\mathfrak{Z})\cup^{\mathfrak{Z}}\bigcup^{\mathfrak{Z}}(\operatorname{atoms}^{\mathfrak{A}}b\cap\mathfrak{Z})$ (used the theorem 3). Again using the theorem 50 from [1], we get

$\operatorname{Cor}'(a\cup^{\mathfrak{A}}b)=\bigcup^{\mathfrak{Z}}\{x\mid x\text{ is an atom of }\mathfrak{Z},\,x\subseteq a\}\cup^{\mathfrak{Z}}\bigcup^{\mathfrak{Z}}\{x\mid x\text{ is an atom of }\mathfrak{Z},\,x\subseteq b\}=\operatorname{Cor}'a\cup^{\mathfrak{Z}}\operatorname{Cor}'b$ (again used the proposition 34 from [1]). □

Bibliography

[1] Victor Porton. Filters on posets and generalizations. *International Journal of Pure and Applied Mathematics*, 74(1):55–119, 2012.