Definition 74 *We will denote $A/(\sim) = A/((\sim) \cap A \times A)$ for a set A and an equivalence relation $\sim$ on a set $B \supseteq A$. I will call $\sim$ a congruence on A when $(\sim) \cap A \times A$ is a congruence on A.*

Theorem 72 *Let $\mathfrak{F}$ be the set of filters over a boolean lattice $\mathfrak{A}$ and $\mathcal{A} \in \mathfrak{F}$. Consider the function $\gamma : Z(D\mathcal{A}) \to \mathfrak{A}/\sim$ defined by the formula (for every $p \in Z(D\mathcal{A})$)*

$$\gamma p = \left\{ X \in \mathfrak{A} \mid X \cap^{\mathfrak{F}} \mathcal{A} = p \right\}.$$

Then:

1. *γ is a lattice isomorphism.*

2. *$\forall Q \in q : \gamma^{-1} q = Q \cap^{\mathfrak{F}} \mathcal{A}$ for every $q \in \mathfrak{A}/\sim$.*

Proof $\forall p \in Z(D\mathcal{A}) : \gamma p \neq \emptyset$ because of the theorem 59. Thus easy to see that $\gamma p \in \mathfrak{A}/\sim$ and that γ is an injection.

Let's prove that γ is a lattice homomorphism:
$\gamma(p_0 \cap^{\mathfrak{F}} p_1) = \left\{ X \in \mathfrak{A} \mid X \cap^{\mathfrak{F}} \mathcal{A} = p_0 \cap^{\mathfrak{F}} p_1 \right\}$;

$$\begin{aligned}
\gamma p_0 \cap^{\mathfrak{A}/\sim} \gamma p_1 &= \\
\left\{ X_0 \in \mathfrak{A} \mid X_0 \cap^{\mathfrak{F}} \mathcal{A} = p_0 \right\} \cap^{\mathfrak{A}/\sim} \left\{ X_1 \in \mathfrak{A} \mid X_1 \cap^{\mathfrak{F}} \mathcal{A} = p_1 \right\} &= \\
\left\{ X_0 \cap^{\mathfrak{F}} X_1 \mid X_0, X_1 \in \mathfrak{A}, X_0 \cap^{\mathfrak{F}} \mathcal{A} = p_0 \wedge X_1 \cap^{\mathfrak{F}} \mathcal{A} = p_1 \right\} &\subseteq \\
\left\{ X' \in \mathfrak{A} \mid X' \cap^{\mathfrak{F}} \mathcal{A} = p_0 \cap^{\mathfrak{F}} p_1 \right\} &= \\
\gamma(p_0 \cap^{\mathfrak{F}} p_1).
\end{aligned}$$

Because $\gamma p_0 \cap^{\mathfrak{A}/\sim} \gamma p_1$ and $\gamma(p_0 \cap^{\mathfrak{F}} p_1)$ are equivalence classes, thus follows $\gamma p_0 \cap^{\mathfrak{A}/\sim} \gamma p_1 = \gamma(p_0 \cap^{\mathfrak{F}} p_1)$.

$\gamma(p_0 \cup^{\mathfrak{F}} p_1) = \left\{ X \in \mathfrak{A} \mid X \cap^{\mathfrak{F}} \mathcal{A} = p_0 \cup^{\mathfrak{F}} p_1 \right\}$;

$$\begin{aligned}
\gamma p_0 \cup^{\mathfrak{A}/\sim} \gamma p_1 &= \\
\left\{ X_0 \in \mathfrak{A} \mid X_0 \cap^{\mathfrak{F}} \mathcal{A} = p_0 \right\} \cup^{\mathfrak{A}/\sim} \left\{ X_1 \in \mathfrak{A} \mid X_1 \cap^{\mathfrak{F}} \mathcal{A} = p_1 \right\} &= \\
\left\{ X_0 \cup^{\mathfrak{F}} X_1 \mid X_0, X_1 \in \mathfrak{A}, X_0 \cap^{\mathfrak{F}} \mathcal{A} = p_0 \wedge X_1 \cap^{\mathfrak{F}} \mathcal{A} = p_1 \right\} &\subseteq \\
\left\{ X_0 \cup^{\mathfrak{F}} X_1 \mid X_0, X_1 \in \mathfrak{A}, (X_0 \cup^{\mathfrak{F}} X_1) \cap^{\mathfrak{F}} \mathcal{A} = p_0 \cup^{\mathfrak{F}} p_1 \right\} &\subseteq \\
\left\{ X' \in \mathfrak{A} \mid X' \cap^{\mathfrak{F}} \mathcal{A} = p_0 \cup^{\mathfrak{F}} p_1 \right\} &= \\
\gamma(p_0 \cup^{\mathfrak{F}} p_1).
\end{aligned}$$

Because $\gamma p_0 \cup^{\mathfrak{A}/\sim} \gamma p_1$ and $\gamma(p_0 \cup^{\mathfrak{F}} p_1)$ are equivalence classes, thus follows $\gamma p_0 \cup^{\mathfrak{A}/\sim} \gamma p_1 = \gamma(p_0 \cup^{\mathfrak{F}} p_1)$.

To finish the proof it's enough to show that $\forall Q \in q : q = \gamma(Q \cap^{\mathfrak{F}} \mathcal{A})$ for every $q \in \mathfrak{A}/\sim$. (From this follows that γ is surjective because q is not empty and thus $\exists Q \in q : q = \gamma(Q \cap^{\mathfrak{F}} \mathcal{A})$.) Really,

$$\gamma(Q \cap^{\mathfrak{F}} \mathcal{A}) = \left\{ X \in \mathfrak{A} \mid X \cap^{\mathfrak{F}} \mathcal{A} = Q \cap^{\mathfrak{F}} \mathcal{A} \right\} = [Q] = q.$$