Theorem 65 *Let* $(\mathfrak{A};\mathfrak{Z})$ *be a semifiltered down-aligned filtrator with finitely meet-closed core* $\mathfrak{Z}$ *which is a complete atomistic lattice and* $\mathfrak{A}$ *is a distributive lattice, then* $\operatorname{Cor}'(a \cup^{\mathfrak{A}} b) = \operatorname{Cor}' a \cup^{\mathfrak{Z}} \operatorname{Cor}' b$ *for every* $a, b \in \mathfrak{A}$.

Proof $\operatorname{Cor}'(a \cup^{\mathfrak{A}} b) = \bigcup^{\mathfrak{Z}} \left\{ x \mid x \text{ is an atom of } \mathfrak{Z}, x \subseteq a \cup^{\mathfrak{A}} b \right\}$ (used proposition 34).

By the theorem 50 we have $\operatorname{Cor}'(a \cup^{\mathfrak{A}} b) = \bigcup^{\mathfrak{Z}}(\operatorname{atoms}^{\mathfrak{A}}(a \cup^{\mathfrak{A}} b) \cap \mathfrak{Z}) = \bigcup^{\mathfrak{Z}}((\operatorname{atoms}^{\mathfrak{A}} a \cup \operatorname{atoms}^{\mathfrak{A}} b) \cap \mathfrak{Z}) = \bigcup^{\mathfrak{Z}}((\operatorname{atoms}^{\mathfrak{A}} a \cap \mathfrak{Z}) \cup (\operatorname{atoms}^{\mathfrak{A}} b \cap \mathfrak{Z})) = \bigcup^{\mathfrak{Z}}(\operatorname{atoms}^{\mathfrak{A}} a \cap \mathfrak{Z}) \cup^{\mathfrak{Z}} \bigcup^{\mathfrak{Z}}(\operatorname{atoms}^{\mathfrak{A}} b \cap \mathfrak{Z})$ (used the theorem 1). Again using theorem 50, we get $\operatorname{Cor}'(a \cup^{\mathfrak{A}} b) = \bigcup^{\mathfrak{Z}} \{x \mid x \text{ is an atom of } \mathfrak{Z}, x \subseteq a\} \cup^{\mathfrak{Z}} \bigcup^{\mathfrak{Z}} \{x \mid x \text{ is an atom of } \mathfrak{Z}, x \subseteq b\} = \operatorname{Cor}' a \cup^{\mathfrak{Z}} \operatorname{Cor}' b$ (again used proposition 34). □

Theorem 66 *Let* $(\mathfrak{F};\mathfrak{A})$ *be a primary filtrator over a complete boolean lattice. Then* $(a \cap^{\mathfrak{F}} b)^+ = a^+ \cup^{\mathfrak{A}} b^+$ *for every* $a, b \in \mathfrak{F}$.

Proof $(\mathfrak{F};\mathfrak{A})$ is a filtered up-aligned complete lattice filtrator with finitely join-closed (theorem 23) co-separable core (theorem 38) which is a complete boolean lattice. Thus by the theorem 60

$$(a \cap^{\mathfrak{F}} b)^+ = \overline{\operatorname{Cor}(a \cap^{\mathfrak{F}} b)} = \overline{\operatorname{Cor} a \cap^{\mathfrak{A}} \operatorname{Cor} b} = \overline{\operatorname{Cor} a} \cup^{\mathfrak{A}} \overline{\operatorname{Cor} b} = a^+ \cup^{\mathfrak{A}} b^+.$$

□

Theorem 67 *Let* $(\mathfrak{A};\mathfrak{Z})$ *be a filtered distributive down-aligned, complete lattice filtrator with finitely meet-closed, separable core which is a complete atomistic boolean lattice. Then* $(a \cup^{\mathfrak{A}} b)^* = a^* \cap^{\mathfrak{Z}} b^*$ *for every* $a, b \in \mathfrak{A}$.

Proof $(a \cup^{\mathfrak{A}} b)^* = \overline{\operatorname{Cor}'(a \cup^{\mathfrak{A}} b)} = \overline{\operatorname{Cor}' a \cup^{\mathfrak{Z}} \operatorname{Cor}' b} = \overline{\operatorname{Cor}' a} \cap^{\mathfrak{Z}} \overline{\operatorname{Cor}' b} = a^* \cap^{\mathfrak{Z}} b^*$ (used the theorem 61). □

Theorem 68 *Let* $\mathfrak{A}$ *be a complete boolean lattice. Then* $(a \cap^{\mathfrak{F}} b)^* = a^* \cup^{\mathfrak{A}} b^*$ *for every* $a, b \in \mathfrak{F}$.

Proof $(\mathfrak{F};\mathfrak{A})$ is a filtered complete lattice filtrator with down-aligned, up-aligned, finitely meet-closed, separable core which is a complete boolean lattice. So

$$(a \cap^{\mathfrak{F}} b)^* = \overline{\operatorname{Cor}(a \cap^{\mathfrak{F}} b)} = \overline{\operatorname{Cor} a \cap^{\mathfrak{A}} \operatorname{Cor} b} = \overline{\operatorname{Cor} a} \cup^{\mathfrak{A}} \overline{\operatorname{Cor} b} = a^* \cup^{\mathfrak{A}} b^*$$

(used the theorem 61). □