

Definition 70 The *edge part* of an element $a \in \mathfrak{A}$ is $\text{Edg } a = a \setminus \text{Cor } a$, the *dual edge part* is $\text{Edg}' a = a \setminus \text{Cor}' a$.

Proposition 33 For a primary filtrator over a complete boolean lattice both edge part and dual edge part are always defined.

Proof Using the theorem 39. □

Knowing core part and edge part or dual core part and dual edge part of a filter object, the filter object can be restored by the formulas:

$$a = \text{Cor } a \cup^{\mathfrak{A}} \text{Edg } a \text{ and } a = \text{Cor}' a \cup^{\mathfrak{A}} \text{Edg}' a.$$

13.1. Core part and atomic elements

Proposition 34 Let $(\mathfrak{A}; \mathfrak{F})$ be a filtrator with join-closed core and \mathfrak{F} is an atomistic lattice. Then for every $a \in \mathfrak{A}$ such that $\text{Cor}' a$ exists we have

$$\text{Cor}' a = \bigcup^{\mathfrak{F}} \{x \mid x \text{ is an atom of } \mathfrak{F}, x \subseteq a\}.$$

Proof

$$\begin{aligned} \text{Cor}' a &= \\ \bigcup^{\mathfrak{F}} \{A \in \mathfrak{F} \mid A \subseteq a\} &= \\ \bigcup^{\mathfrak{F}} \left\{ \bigcup^{\mathfrak{F}} \text{atoms}^{\mathfrak{F}} A \mid A \in \mathfrak{F}, A \subseteq a \right\} &= \\ \bigcup^{\mathfrak{F}} \bigcup \{ \text{atoms}^{\mathfrak{F}} A \mid A \in \mathfrak{F}, A \subseteq a \} &= \\ \bigcup^{\mathfrak{F}} \{x \mid x \text{ is an atom of } \mathfrak{F}, x \subseteq a\}. & \end{aligned}$$

□

14. Distributivity of core part over lattice operations

Theorem 64 If $(\mathfrak{A}; \mathfrak{F})$ is a join-closed filtrator and \mathfrak{A} is a meet-semilattice and \mathfrak{F} is a complete lattice, then

$$\text{Cor}'(a \cap^{\mathfrak{A}} b) = \text{Cor}' a \cap^{\mathfrak{F}} \text{Cor}' b.$$

Proof From theorem conditions follows that $\text{Cor}'(a \cap^{\mathfrak{A}} b)$ exists.

We have $\text{Cor}' p \subseteq p$ for every $p \in \mathfrak{A}$ because our filtrator is with join-closed core.

Obviously $\text{Cor}'(a \cap^{\mathfrak{A}} b) \subseteq \text{Cor}' a$ and $\text{Cor}'(a \cap^{\mathfrak{A}} b) \subseteq \text{Cor}' b$.

If $x \subseteq \text{Cor}' a$ and $x \subseteq \text{Cor}' b$ for some $x \in \mathfrak{F}$ then $x \subseteq a$ and $x \subseteq b$, thus $x \subseteq a \cap^{\mathfrak{A}} b$ and $x \subseteq \text{Cor}'(a \cap^{\mathfrak{A}} b)$. □