Proof Our filtrator is with join-closed core. $a^* = \bigcup^{\mathfrak{A}} \left\{ c \in \mathfrak{A} \mid c \cap^{\mathfrak{A}} a = 0 \right\}$. But $c \cap^{\mathfrak{A}} a = 0 \Rightarrow \exists C \in \operatorname{up} c : C \cap^{\mathfrak{A}} a = 0$. So

$$\begin{aligned} a^* &= \\ \bigcup^{\mathfrak{A}} \left\{ C \in \mathfrak{Z} \mid C \cap^{\mathfrak{A}} a = 0 \right\} &= \\ \bigcup^{\mathfrak{A}} \left\{ C \in \mathfrak{Z} \mid a \subseteq \overline{C} \right\} &= \\ \bigcup^{\mathfrak{A}} \left\{ \overline{C} \mid C \in \mathfrak{Z}, a \subseteq C \right\} &= \\ \bigcup^{\mathfrak{A}} \left\{ \overline{C} \mid C \in \operatorname{up} a \right\} &= \\ \bigcup^{\mathfrak{Z}} \left\{ \overline{C} \mid C \in \operatorname{up} a \right\} &= \\ \overline{\bigcap^{\mathfrak{Z}} \left\{ C \mid C \in \operatorname{up} a \right\}} &= \\ \overline{\bigcap^{\mathfrak{Z}} \operatorname{up} a} &= \\ \overline{\operatorname{Cor} a}. \end{aligned}$$

(used the theorem 27).

$\operatorname{Cor} a = \operatorname{Cor}' a$ by the theorem 26. □

Corollary 19 *If $(\mathfrak{A}; \mathfrak{Z})$ is filtered down-aligned and up-aligned complete lattice filtrator with finitely meet-closed, separable and co-separable core which is a complete boolean lattice, then $a^* = a^+$ for every $a \in \mathfrak{A}$.*

Proof Comparing two last theorems. □

Theorem 62 *If $(\mathfrak{A}; \mathfrak{Z})$ is a complete lattice filtrator with join-closed separable core which is a complete lattice, then $a^* \in \mathfrak{Z}$ for every $a \in \mathfrak{A}$.*

Proof $\left\{ c \in \mathfrak{A} \mid c \cap^{\mathfrak{A}} a = 0 \right\} \supseteq \left\{ A \in \mathfrak{Z} \mid A \cap^{\mathfrak{A}} a = 0 \right\}$; consequently $a^* \supseteq \bigcup^{\mathfrak{A}} \left\{ A \in \mathfrak{Z} \mid A \cap^{\mathfrak{A}} a = 0 \right\}$.

But if $c \in \left\{ c \in \mathfrak{A} \mid c \cap^{\mathfrak{A}} a = 0 \right\}$ then exists $A \in \mathfrak{Z}$ such that $A \supseteq c$ and $A \cap^{\mathfrak{A}} a = 0$ that is $A \in \left\{ A \in \mathfrak{Z} \mid A \cap^{\mathfrak{A}} a = 0 \right\}$. Consequently $a^* \subseteq \bigcup^{\mathfrak{A}} \left\{ A \in \mathfrak{Z} \mid A \cap^{\mathfrak{A}} a = 0 \right\}$.

We have $a^* = \bigcup^{\mathfrak{A}} \left\{ A \in \mathfrak{Z} \mid A \cap^{\mathfrak{A}} a = 0 \right\} = \bigcup^{\mathfrak{Z}} \left\{ A \in \mathfrak{Z} \mid A \cap^{\mathfrak{A}} a = 0 \right\} \in \mathfrak{Z}$. □

Theorem 63 *If $(\mathfrak{A}; \mathfrak{Z})$ is an up-aligned filtered complete lattice filtrator co-separable core which is a complete boolean lattice, then a^+ is dual pseudocomplement of a, that is $a^+ = \min \left\{ c \in \mathfrak{A} \mid c \cup^{\mathfrak{A}} a = 1 \right\}$ for every $a \in \mathfrak{A}$.*

Proof Our filtrator is with join-closed core. It's enough to prove that $a^+ \cup^{\mathfrak{A}} a = 1$. But $a^+ \cup^{\mathfrak{A}} a = \overline{\operatorname{Cor} a} \cup^{\mathfrak{A}} a \supseteq \overline{\operatorname{Cor} a} \cup^{\mathfrak{A}} \operatorname{Cor} a = \overline{\operatorname{Cor} a} \cup^{\mathfrak{Z}} \operatorname{Cor} a = 1$ (used the theorem 24 and the fact that our filtrator is filtered). □