Proposition 32 *$(a \cup b) \setminus^{*} b \subseteq a$ for an arbitrary complete lattice.*

Proof $(a \cup b) \setminus^{*} b = \bigcap \{z \in \mathfrak{A} \mid a \cup b \subseteq b \cup z\}$.

But $a \subseteq z \Rightarrow a \cup b \subseteq b \cup z$. So $\{z \in \mathfrak{A} \mid a \cup b \subseteq b \cup z\} \supseteq \{z \in \mathfrak{A} \mid a \subseteq z\}$.

Consequently, $(a \cup b) \setminus^{*} b \subseteq \bigcap \{z \in \mathfrak{A} \mid a \subseteq z\} = a$. □

13. Complements and core parts

Lemma 6 *If $(\mathfrak{A}; \mathfrak{Z})$ is a filtered, up-aligned filtrator with co-separable core which is a complete lattice, then for any $a, c \in \mathfrak{A}$*

$$c \equiv^{\mathfrak{A}} a \Leftrightarrow c \equiv^{\mathfrak{A}} \operatorname{Cor} a.$$

Proof

⇒ If $c \equiv^{\mathfrak{A}} a$ then by co-separability of the core exists $K \in \operatorname{down} a$ such that $c \equiv^{\mathfrak{A}} K$. To finish the proof we will show that $K \subseteq \operatorname{Cor} a$. To show this is enough to show that $\forall X \in \operatorname{up} a : K \subseteq X$ what is obvious.

⇐ Because $\operatorname{Cor} a \subseteq a$ (by the theorem 24 using that our filtrator is filtered).

□

Theorem 60 *If $(\mathfrak{A}; \mathfrak{Z})$ is a filtered up-aligned complete lattice filtrator with co-separable core which is a complete boolean lattice, then $a^{+} = \overline{\operatorname{Cor} a}$ for every $a \in \mathfrak{A}$.*

Proof Our filtrator is with join-closed core.

$$\begin{aligned} a^{+} &= \\ \bigcap\nolimits^{\mathfrak{A}} \left\{c \in \mathfrak{A} \mid c \cup^{\mathfrak{A}} a = 1\right\} &= \\ \bigcap\nolimits^{\mathfrak{A}} \left\{c \in \mathfrak{A} \mid c \cup^{\mathfrak{A}} \operatorname{Cor} a = 1\right\} &= \\ \bigcap\nolimits^{\mathfrak{A}} \left\{c \in \mathfrak{A} \mid c \supseteq \overline{\operatorname{Cor} a}\right\} &= \\ \overline{\operatorname{Cor} a}. \end{aligned}$$

(used the lemma and the theorem 27). □

Corollary 18 *If $(\mathfrak{A}; \mathfrak{Z})$ is a filtered up-aligned complete lattice filtrator with co-separable core which is a complete boolean lattice, then $a^{+} \in \mathfrak{Z}$ for every $a \in \mathfrak{A}$.*

Theorem 61 *If $(\mathfrak{A}; \mathfrak{Z})$ is a filtered complete lattice filtrator with down-aligned, finitely meet-closed, separable core which is a complete boolean lattice, then $a^{*} = \overline{\operatorname{Cor} a} = \overline{\operatorname{Cor}' a}$.*