Theorem 57 *$a \setminus^* b = \bigcap \{ z \in \mathfrak{A} \mid z \subseteq a \wedge a \subseteq b \cup z \}$ where $\mathfrak{A}$ is a distributive lattice and $a, b \in \mathfrak{A}$.*

Proof Obviously $\{ z \in \mathfrak{A} \mid z \subseteq a \wedge a \subseteq b \cup z \} \subseteq \{ z \in \mathfrak{A} \mid a \subseteq b \cup z \}$. Thus $\bigcap \{ z \in \mathfrak{A} \mid z \subseteq a \wedge a \subseteq b \cup z \} \supseteq a \setminus^* b$.

Let $z \in \mathfrak{A}$ and $z' = z \cap a$.

$a \subseteq b \cup z \Rightarrow a \subseteq (b \cup z) \cap a \Leftrightarrow a \subseteq (b \cap a) \cup (z \cap a) \Leftrightarrow a \subseteq (b \cap a) \cup z' \Rightarrow a \subseteq b \cup z'$ and $a \subseteq b \cup z \Leftarrow a \subseteq b \cup z'$. Thus $a \subseteq b \cup z \Leftrightarrow a \subseteq b \cup z'$.

If $z \in \{ z \in \mathfrak{A} \mid a \subseteq b \cup z \}$ then $a \subseteq b \cup z'$ and thus $z' \in \{ z \in \mathfrak{A} \mid z \subseteq a \wedge a \subseteq b \cup z \}$. But $z' \subseteq z$ thus having $\bigcap \{ z \in \mathfrak{A} \mid z \subseteq a \wedge a \subseteq b \cup z \} \subseteq \bigcap \{ z \in \mathfrak{A} \mid a \subseteq b \cup z \}$.

□

Remark 12 If we drop the requirement that $\mathfrak{A}$ is distributive, two formulas for quasidifference (the definition and the last theorem) fork.

Obvious 21 *Dual quasicomplement is the dual of quasicomplement.*

Obvious 22

- *Every pseudocomplement is quasicomplement.*
- *Every dual pseudocomplement is dual quasicomplement.*
- *Every pseudodifference is quasidifference.*

Below we will stick to the more general quasies than pseudos. If needed, one can check that a quasicomplement a^* is a pseudocomplement by the equation $a^* \asymp a$ (and analogously with other quasies).

Next we will express quasidifference through quasicomplement.

Proposition 31

1. *$a \setminus^* b = a \setminus^* (a \cap b)$ for any distributive lattice;*
2. *$a \# b = a \# (a \cap b)$ for any distributive lattice with least element.*

Proof

1. $a \subseteq (a \cap b) \cup z \Leftrightarrow a \subseteq (a \cup z) \cap (b \cup z) \Leftrightarrow a \subseteq a \cup z \wedge a \subseteq b \cup z \Leftrightarrow a \subseteq b \cup z$. Thus $a \setminus^* (a \cap b) = \bigcap \{ z \in \mathfrak{A} \mid a \subseteq (a \cap b) \cup z \} = \bigcap \{ z \in \mathfrak{A} \mid a \subseteq b \cup z \} = a \setminus^* b$.
2. $a \# (a \cap b) = \bigcup \{ z \in \mathfrak{A} \mid z \subseteq a \wedge z \cap a \cap b = 0 \} = \bigcup \{ z \in \mathfrak{A} \mid z \subseteq a \wedge (z \cap a) \cap a \cap b = 0 \} = \bigcup \{ z \cap a \mid z \in \mathfrak{A}, z \cap a \cap b = 0 \} = \bigcup \{ z \in \mathfrak{A} \mid z \subseteq a, z \cap b = 0 \} = a \# b$.

□

I will denote Da the lattice $\{ x \in \mathfrak{A} \mid x \subseteq a \}$.

Theorem 58 *For $a, b \in \mathfrak{A}$ where $\mathfrak{A}$ is a distributive lattice with least element*

1. *$a \setminus^* b = (a \cap b)^{+(Da)}$;*