$b \subseteq b_\beta$ and so $\forall \beta \in \operatorname{card} T \forall \alpha \in \beta : b \subseteq a_\alpha$. Let $\alpha \in \operatorname{card} T$. Then (because $\operatorname{card} A$ is limit ordinal, see [15]) exist $\beta \in \operatorname{card} T$ such that $\alpha \in \beta \in \operatorname{card} T$. So $b \subseteq a_\alpha$ for every $\alpha \in \operatorname{card} T$. Thus $b \subseteq \bigcap T$.

Finally $\bigcap T = b \in S$.

□

Theorem 56 *Let $\mathfrak{A}$ be a boolean lattice. For any $S \in \mathcal{P}\mathfrak{F}$ the condition $\exists \mathcal{F} \in \mathfrak{F} : S = \star \mathcal{F}$ is equivalent to conjunction of the following items:*

1. *S is a free star on $\mathfrak{F}$;*

2. *S is filter-closed.*

Proof

$\Rightarrow$ 1. That $0 \notin \star\mathcal{F}$ is obvious. For every $a, b \in \mathfrak{F}$

$$\begin{aligned} a \cup^{\mathfrak{F}} b \in \star\mathcal{F} &\Leftrightarrow \\ (a \cup^{\mathfrak{F}} b) \cap^{\mathfrak{F}} \mathcal{F} \neq 0 &\Leftrightarrow \\ (a \cap^{\mathfrak{F}} \mathcal{F}) \cup (b \cap^{\mathfrak{F}} \mathcal{F}) \neq 0 &\Leftrightarrow \\ a \cap^{\mathfrak{F}} \mathcal{F} \neq 0 \vee b \cap^{\mathfrak{F}} \mathcal{F} \neq 0 &\Leftrightarrow \\ a \in \star S \vee b \in \star\mathcal{F}. & \end{aligned}$$

(taken into account the corollary 10). So $\star\mathcal{F}$ is a free star on $\mathfrak{F}$.

2. We have $T \subseteq S$ and need to prove that $\bigcap^{\mathfrak{F}} T \cap \mathcal{F} \neq 0$. Because $\langle \mathcal{F} \cap^{\mathfrak{F}} \rangle T$ is a generalized filter base, $0 \in \langle \mathcal{F} \cap^{\mathfrak{F}} \rangle T \Leftrightarrow \bigcap^{\mathfrak{F}} \langle \mathcal{F} \cap^{\mathfrak{F}} \rangle T = 0 \Leftrightarrow \bigcap^{\mathfrak{F}} T \cap^{\mathfrak{F}} \mathcal{F} = 0$. So it's left to prove $0 \notin \langle \mathcal{F} \cap^{\mathfrak{F}} \rangle T$ what follows from $T \subseteq S$.

$\Leftarrow$ Let S be a free star on $\mathfrak{F}$. Then for every $A, B \in \mathfrak{A}$

$$\begin{aligned} A, B \in S \cap \mathfrak{A} &\Leftrightarrow \\ A, B \in S &\Leftrightarrow \\ A \cup^{\mathfrak{F}} B \in S &\Leftrightarrow \\ A \cup^{\mathfrak{A}} B \in S &\Leftrightarrow \\ A \cup^{\mathfrak{A}} B \in S \cap \mathfrak{A} & \end{aligned}$$

(taken into account the theorem 23). So $S \cap \mathfrak{A}$ is a free star on $\mathfrak{A}$.

Thus there exists $\mathcal{F} \in \mathfrak{F}$ such that $\partial\mathcal{F} = S \cap \mathfrak{A}$. We have $\operatorname{up} \mathcal{X} \subseteq S \Leftrightarrow \mathcal{X} \in S$ (because S is filter-closed) for every $\mathcal{X} \in \mathfrak{F}$; then (taking in account