Then

$$F = \bigcup^{\mathfrak{A}} \left\{ K_L \mid L \in \mathfrak{Z}, \mathcal{F} \cap^{\mathfrak{A}} L \neq 0 \right\}.$$

Obviously $F \subseteq \mathcal{F}$. We have $L \cap^{\mathfrak{A}} \mathcal{F} \neq 0 \Rightarrow L \cap^{\mathfrak{Z}} \mathcal{F} \neq 0 \Rightarrow L \cap^{\mathfrak{A}} \mathcal{F} \neq 0 \Rightarrow K_L \cap^{\mathfrak{Z}} F \neq 0 \Rightarrow L \cap^{\mathfrak{Z}} F \neq 0$, thus by star separability of our filtrator $\mathcal{F} \subseteq F$ and so $\mathcal{F} = F \in \mathfrak{Z}$.

□

Theorem 54 *If $\mathfrak{A}$ is a complete boolean lattice then for each $\mathcal{F} \in \mathfrak{F}$*

$$\mathcal{F} \in \mathfrak{A} \Leftrightarrow \forall S \in \mathscr{P}\mathfrak{A} : \left(\bigcup^{\mathfrak{A}} S \in \partial \mathcal{F} \Rightarrow S \cap \partial \mathcal{F} \neq \emptyset \right).$$

Proof

$$\begin{aligned}
\forall S \in \mathscr{P}\mathfrak{A} : \left(\bigcup^{\mathfrak{A}} S \in \partial \mathcal{F} \Rightarrow S \cap \partial \mathcal{F} \neq \emptyset \right) &\Leftrightarrow \\
\forall S \in \mathscr{P}\mathfrak{A} : \left(\bigcup^{\mathfrak{A}} S \notin \partial \mathcal{F} \Leftarrow S \cap \partial \mathcal{F} = \emptyset \right) &\Leftrightarrow \\
\forall S \in \mathscr{P}\mathfrak{A} : \left(\overline{\bigcup^{\mathfrak{A}} S} \in \operatorname{up} \mathcal{F} \Leftarrow \langle \neg \rangle S \subseteq \operatorname{up} \mathcal{F} \right) &\Leftrightarrow \\
\forall S \in \mathscr{P}\mathfrak{A} : \left(\bigcap^{\mathfrak{A}} S \in \operatorname{up} \mathcal{F} \Leftarrow S \subseteq \operatorname{up} \mathcal{F} \right),
\end{aligned}$$

but

$$\begin{aligned}
\mathcal{F} \in \mathfrak{A} &\Rightarrow \\
\forall S \in \mathscr{P}\mathfrak{A} : \left(\bigcap^{\mathfrak{A}} S \in \operatorname{up} \mathcal{F} \Leftarrow S \subseteq \operatorname{up} \mathcal{F} \right) &\Rightarrow \\
\bigcap^{\mathfrak{A}} \operatorname{up} \mathcal{F} \in \operatorname{up} \mathcal{F} &\Rightarrow \\
\mathcal{F} \in \mathfrak{A}.
\end{aligned}$$

□

Definition 64 *Let S be a subset of a meet-semilattice. The **filter base generated by** S is the set*

$$[S]_\cap \stackrel{\text{def}}{=} \{ a_0 \cap \ldots \cap a_n \mid a_i \in S, i = 0, 1, \ldots \}.$$

Lemma 4 *The set of all finite subsets of an infinite set A has the same cardinality as A.*

Proof Let denote the number of n-element subsets of A as s_n. Obviously $s_n \leqslant \operatorname{card} A^n = \operatorname{card} A$. Then the number S of all finite subsets of A is equal to $s_0 + s_1 + \ldots \leqslant \operatorname{card} A + \operatorname{card} A + \ldots = \operatorname{card} A$. That $S \geqslant \operatorname{card} A$ is obvious. So $S = \operatorname{card} A$. □