Proof Because (used the theorem 20) $\mathfrak{F}$ is atomic (the theorem 47) and separable. □

Corollary 17 *If $\mathfrak{A}$ is a boolean lattice then $\mathfrak{F}$ is atomically separable.*

Proof By the theorem 14. □

Theorem 49 *When the base poset $\mathfrak{A}$ is a boolean lattice, then the filtrator $(\mathfrak{F};\mathfrak{A})$ is central.*

Proof We can conclude that $\mathfrak{F}$ is atomically separable (the corollary 17) and with separable core (the theorem 37).

We need to prove that $Z(\mathfrak{F}) = \mathfrak{A}$.

Let $\mathcal{X} \in Z(\mathfrak{F})$. Then exists $\mathcal{Y} \in Z(\mathfrak{F})$ such that $\mathcal{X} \cap^{\mathfrak{F}} \mathcal{Y} = 0$ and $\mathcal{X} \cup^{\mathfrak{F}} \mathcal{Y} = 1$. Consequently there are $X \in \operatorname{up} \mathcal{X}$ such that $X \cap^{\mathfrak{F}} \mathcal{Y} = 0$; we have also $X \cup^{\mathfrak{F}} \mathcal{Y} = 1$. Suppose $X \supset \mathcal{X}$. Then exists $a \in \operatorname{atoms}^{\mathfrak{F}} X$ such that $a \notin \operatorname{atoms}^{\mathfrak{F}} \mathcal{X}$. We can conclude also $a \notin \operatorname{atoms}^{\mathfrak{F}} \mathcal{Y}$ (otherwise $X \cap^{\mathfrak{F}} \mathcal{Y} \neq 0$). Thus $a \notin \operatorname{atoms}^{\mathfrak{F}}(\mathcal{X} \cup^{\mathfrak{F}} \mathcal{Y})$ and consequently $\mathcal{X} \cup^{\mathfrak{F}} \mathcal{Y} \neq 1$ what is a contradiction. We have $\mathcal{X} = X \in \mathfrak{A}$.

Let now $X \in \mathfrak{A}$. Let $Y = 1 \setminus^{\mathfrak{A}} X$. We have $X \cap^{\mathfrak{A}} Y = 0$ and $X \cup^{\mathfrak{A}} Y = 1$. Thus $X \cap^{\mathfrak{F}} Y = \bigcap^{\mathfrak{A}} \left\{ X \cap^{\mathfrak{A}} Y \right\} = 0$; $X \cup^{\mathfrak{F}} Y = \bigcap^{\mathfrak{F}}(\operatorname{up} X \cap \operatorname{up} Y) = \bigcap^{\mathfrak{F}} \{1\} = 1$. We have shown that $X \in Z(\mathfrak{F})$. □

10. Atomic filter objects

See [2] and [4] for more detailed treatment of ultrafilters and prime filters.

Theorem 50 *Let $(\mathfrak{A};\mathfrak{Z})$ be a semifiltered down-aligned filtrator with finitely meet-closed core $\mathfrak{Z}$ which is a meet-semilattice. Then a is an atom of $\mathfrak{Z}$ iff $a \in \mathfrak{Z}$ and a is an atom of $\mathfrak{A}$.*

Proof

⇐ Obvious.

⇒ We need to prove that if a is an atom of $\mathfrak{Z}$ then a is an atom of $\mathfrak{A}$. Suppose the contrary that a is not an atom of $\mathfrak{A}$. Then exists $x \in \mathfrak{A}$ such that $0 \neq x \subset a$. Because "up" is a straight monotone map from $\mathfrak{A}$ to the dual of the poset $\mathcal{P}\mathfrak{Z}$ (the theorem 10), $\operatorname{up} a \subset \operatorname{up} x$. So exists $K \in \operatorname{up} x$ such that $K \notin \operatorname{up} a$. Also $a \in \operatorname{up} x$. We have $K \cap^{\mathfrak{Z}} a = K \cap^{\mathfrak{A}} a \in \operatorname{up} x$; $K \cap^{\mathfrak{Z}} a \neq 0$ and $K \cap^{\mathfrak{Z}} a \subset a$. So a is not an atom of $\mathfrak{Z}$.

□

Theorem 51 *Let $(\mathfrak{A};\mathfrak{Z})$ be a down-aligned semifiltered filtrator and $\mathfrak{A}$ is a meet-semilattice. Then $a \in \mathfrak{A}$ is an atom of $\mathfrak{A}$ iff $\operatorname{up} a = \partial a$.*