Proof For every $A, B \in \mathfrak{Z}$

$$\begin{aligned} A \cup^{\mathfrak{Z}} B \in \partial a &\Leftrightarrow \\ A \cup^{\mathfrak{A}} B \in \partial a &\Leftrightarrow \\ (A \cup^{\mathfrak{A}} B) \cap^{\mathfrak{A}} a \neq 0 &\Leftrightarrow \\ (A \cap^{\mathfrak{A}} a) \cup^{\mathfrak{A}} (B \cap^{\mathfrak{A}} a) \neq 0 &\Leftrightarrow \\ A \cap^{\mathfrak{A}} a \neq 0 \vee B \cap^{\mathfrak{A}} a \neq 0 &\Leftrightarrow \\ A \in \partial a \vee \in \partial a. & \end{aligned}$$

That ∂a doesn't contain 0 is obvious. □

Definition 62 *I call a filtrator **star-separable** when its core is a separation subset of its base.*

9.3. Stars of filters on boolean lattices

In this section we will consider the set of filter objects $\mathfrak{F}$ on a boolean lattice $\mathfrak{A}$.

Theorem 44 *If $\mathfrak{A}$ is a boolean lattice and $\mathcal{A} \in \mathfrak{F}$ then*

1. $\partial \mathcal{A} = \left\{ \overline{X} \mid X \in \mathfrak{A} \setminus \operatorname{up} \mathcal{A} \right\}$*;*

2. $\operatorname{up} \mathcal{A} = \left\{ \overline{X} \mid X \in \mathfrak{A} \setminus \partial \mathcal{A} \right\}$.

Proof 1. For any $K \in \mathfrak{A}$ (taking into account the theorems 29, 37, and 27)

$$\begin{aligned} K \in \left\{ \overline{X} \mid X \in \mathfrak{A} \setminus \operatorname{up} \mathcal{A} \right\} &\Leftrightarrow \\ \overline{K} \in \mathfrak{A} \setminus \operatorname{up} \mathcal{A} &\Leftrightarrow \\ \overline{K} \notin \operatorname{up} \mathcal{A} &\Leftrightarrow \\ \overline{K} \not\supseteq \mathcal{A} &\Leftrightarrow \\ K \not\asymp^{\mathfrak{F}} \mathcal{A} &\Leftrightarrow \\ K \in \partial \mathcal{A}. & \end{aligned}$$

2. For any $K \in \mathfrak{A}$ (taking into account the same theorems)

$$\begin{aligned} K \in \left\{ \overline{X} \mid X \in \mathfrak{A} \setminus \partial \mathcal{A} \right\} &\Leftrightarrow \\ \overline{K} \in \mathfrak{A} \setminus \partial \mathcal{A} &\Leftrightarrow \\ \overline{K} \notin \partial \mathcal{A} &\Leftrightarrow \\ \overline{K} \asymp^{\mathfrak{F}} \mathcal{A} &\Leftrightarrow \\ K \supseteq \mathcal{A} &\Leftrightarrow \\ K \in \operatorname{up} \mathcal{A}. & \end{aligned}$$

□