Theorem 38 *Let $(\mathfrak{A};\mathfrak{Z})$ be an up-aligned filtered filtrator whose core is a meet infinite distributive complete lattice. Then this filtrator is with co-separable core.*

Proof Our filtrator is with join-closed core.

Let $a,b\in\mathfrak{A}$. $\operatorname{Cor} a$ and $\operatorname{Cor} b$ exist since $\mathfrak{Z}$ is a complete lattice.

$\operatorname{Cor} a\in\operatorname{down} a$ and $\operatorname{Cor} b\in\operatorname{down} b$ by the corollary 6 since our filtrator is filtered. So we have

$$\begin{aligned}
\exists x\in\operatorname{down} a, y\in\operatorname{down} b : x\cup^{\mathfrak{A}} y=1 &\Leftarrow\\
\operatorname{Cor} a\cup^{\mathfrak{A}}\operatorname{Cor} b=1 &\Leftrightarrow \quad \text{(by finite join-closedness of the core)}\\
\operatorname{Cor} a\cup^{\mathfrak{Z}}\operatorname{Cor} b=1 &\Leftrightarrow\\
\bigcap\nolimits^{\mathfrak{Z}}\operatorname{up} a\cup^{\mathfrak{Z}}\bigcap\nolimits^{\mathfrak{Z}}\operatorname{up} b=1 &\Leftrightarrow \quad \text{(by infinite distributivity)}\\
\bigcap\nolimits^{\mathfrak{Z}}\left\{x\cup^{\mathfrak{Z}} y \mid x\in\operatorname{up} a, y\in\operatorname{up} b\right\}=1 &\Leftrightarrow\\
\forall x\in\operatorname{up} a, y\in\operatorname{up} b : x\cup^{\mathfrak{Z}} y=1 &\Leftrightarrow \quad \text{(by finite join-closedness of the core)}\\
\forall x\in\operatorname{up} a, y\in\operatorname{up} b : x\cup^{\mathfrak{A}} y=1 &\Leftarrow\\
a\cup^{\mathfrak{A}} b=1.&
\end{aligned}$$

□

7.6. Filters over boolean lattices

Theorem 39 *If $\mathfrak{A}$ is a boolean lattice then $a\setminus^{\mathfrak{F}} B=a\cap^{\mathfrak{F}}\overline{B}$ (where the complement is taken on $\mathfrak{A}$).*

Proof $\mathfrak{F}$ is distributive by the theorem 10. Our filtrator is with finitely meet-closed core by the theorem 29 and with join-closed core by the theorem 23.

$(a\cap^{\mathfrak{F}}\overline{B})\cup^{\mathfrak{F}} B=(a\cup^{\mathfrak{F}} B)\cap^{\mathfrak{F}}(\overline{B}\cup^{\mathfrak{F}} B)=(a\cup^{\mathfrak{F}} B)\cap^{\mathfrak{F}}(\overline{B}\cup^{\mathfrak{A}} B)=(a\cup^{\mathfrak{F}} B)\cap^{\mathfrak{F}} 1=a\cup^{\mathfrak{F}} B$.

$(a\cap^{\mathfrak{F}}\overline{B})\cap^{\mathfrak{F}} B=a\cap^{\mathfrak{F}}(\overline{B}\cap^{\mathfrak{F}} B)=a\cap^{\mathfrak{F}}(\overline{B}\cap^{\mathfrak{A}} B)=a\cap^{\mathfrak{F}} 0=0$.

So $a\cap^{\mathfrak{F}}\overline{B}$ is the difference of a and B. □

7.7. Distributivity for an element of boolean core

Lemma 3 *Let $\mathfrak{F}$ be the set of filter objects over a boolean lattice $\mathfrak{A}$.*

Then $A\cap^{\mathfrak{F}}$ is a lower adjoint of $\overline{A}\cup^{\mathfrak{F}}$ for every $A\in\mathfrak{A}$.

Proof We will use the theorem 8.

That $A\cap^{\mathfrak{F}}$ and $\overline{A}\cup^{\mathfrak{F}}$ are monotone is obvious.

We need to prove (for every $x,y\in\mathfrak{F}$) that

$$x\subseteq\overline{A}\cup^{\mathfrak{F}}(A\cap^{\mathfrak{F}} x)\quad\text{and}\quad A\cap^{\mathfrak{F}}(\overline{A}\cup^{\mathfrak{F}} y)\subseteq y.$$

Really, $\overline{A}\cup^{\mathfrak{F}}(A\cap^{\mathfrak{F}} x)=(\overline{A}\cup^{\mathfrak{F}} A)\cap^{\mathfrak{F}}(\overline{A}\cup^{\mathfrak{F}} x)=(\overline{A}\cup^{\mathfrak{A}} A)\cap^{\mathfrak{F}}(\overline{A}\cup^{\mathfrak{F}} x)=1\cap^{\mathfrak{F}}(\overline{A}\cup^{\mathfrak{F}} x)=\overline{A}\cup^{\mathfrak{F}} x\supseteq x$ and $A\cap^{\mathfrak{F}}(\overline{A}\cup^{\mathfrak{F}} y)=(A\cap^{\mathfrak{F}}\overline{A})\cup^{\mathfrak{F}}(A\cap^{\mathfrak{F}} y)=(A\cap^{\mathfrak{A}}\overline{A})\cup^{\mathfrak{F}}(A\cap^{\mathfrak{F}} y)=0\cup^{\mathfrak{F}}(A\cap^{\mathfrak{F}} y)=A\cap^{\mathfrak{F}} y\subseteq y$. □