Proof Taking in account the previous subsection, we have:

$$\begin{aligned}
\operatorname{up}\left(\mathcal{A} \cup^{\mathfrak{F}} \bigcap{}^{\mathfrak{F}} S\right) &= \\
\operatorname{up}\mathcal{A} \cap \operatorname{up}\bigcap{}^{\mathfrak{F}} S &= \\
\operatorname{up}\mathcal{A} \cap \left\{K_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} K_n \mid K_i \in \bigcup \langle\operatorname{up}\rangle S \text{ where } i = 0, \ldots, n \text{ for } n \in \mathbb{N}\right\} &= \\
\left\{K_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} K_n \mid K_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} K_n \in \operatorname{up}\mathcal{A}, K_i \in \bigcup \langle\operatorname{up}\rangle S \text{ where } i = 0, \ldots, n \text{ for } n \in \mathbb{N}\right\} &= \\
\left\{K_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} K_n \mid K_i \in \operatorname{up}\mathcal{A}, K_i \in \bigcup \langle\operatorname{up}\rangle S \text{ where } i = 0, \ldots, n \text{ for } n \in \mathbb{N}\right\} &= \\
\left\{K_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} K_n \mid K_i \in \operatorname{up}\mathcal{A}, K_i \in \bigcup \{\operatorname{up}\mathcal{X} \mid \mathcal{X} \in S\} \text{ where } i = 0, \ldots, n \text{ for } n \in \mathbb{N}\right\} &= \\
\left\{K_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} K_n \mid K_i \in \operatorname{up}\mathcal{A} \cap \bigcup \{\operatorname{up}\mathcal{X} \mid \mathcal{X} \in S\} \text{ where } i = 0, \ldots, n \text{ for } n \in \mathbb{N}\right\} &= \\
\left\{K_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} K_n \mid K_i \in \bigcup \{\operatorname{up}\mathcal{A} \cap \operatorname{up}\mathcal{X} \mid \mathcal{X} \in S\} \text{ where } i = 0, \ldots, n \text{ for } n \in \mathbb{N}\right\} &= \\
\left\{K_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} K_n \mid K_i \in \bigcup \{\operatorname{up}(\mathcal{A} \cup^{\mathfrak{F}} \mathcal{X}) \mid \mathcal{X} \in S\} \text{ where } i = 0, \ldots, n \text{ for } n \in \mathbb{N}\right\} &= \\
\left\{K_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} K_n \mid K_i \in \bigcup \langle\operatorname{up}\rangle \{\mathcal{A} \cup^{\mathfrak{F}} \mathcal{X} \mid \mathcal{X} \in S\} \text{ where } i = 0, \ldots, n \text{ for } n \in \mathbb{N}\right\} &= \\
\operatorname{up}\bigcap{}^{\mathfrak{F}} \{\mathcal{A} \cup^{\mathfrak{F}} \mathcal{X} \mid \mathcal{X} \in S\}.
\end{aligned}$$

□

Corollary 10 *If $\mathfrak{A}$ is a distributive lattice with greatest element, then $\mathfrak{F}$ is also a distributive lattice.*

Corollary 11 *If $\mathfrak{A}$ is a distributive lattice with greatest element, then $\mathfrak{F}$ is a co-brouwerian lattice.*

7.5. Separability of core for primary filtrators

Theorem 37 *A primary filtrator with least element, whose base is a distributive lattice, is with separable core.*

Proof Let $\mathcal{A} \asymp^{\mathfrak{F}} \mathcal{B}$ where $\mathcal{A}, \mathcal{B} \in \mathfrak{F}$.

$$\operatorname{up}(\mathcal{A} \cap^{\mathfrak{F}} \mathcal{B}) = \left\{A \cap^{\mathfrak{A}} B \mid A \in \operatorname{up}\mathcal{A}, B \in \operatorname{up}\mathcal{B}\right\}.$$

So

$$\begin{aligned}
\mathcal{A} \asymp^{\mathfrak{F}} \mathcal{B} &\Leftrightarrow \\
0 \in \operatorname{up}(\mathcal{A} \cap^{\mathfrak{F}} \mathcal{B}) &\Leftrightarrow \\
\exists A \in \operatorname{up}\mathcal{A}, B \in \operatorname{up}\mathcal{B} : A \cap^{\mathfrak{A}} B = 0 &\Leftrightarrow \\
\exists A \in \operatorname{up}\mathcal{A}, B \in \operatorname{up}\mathcal{B} : A \cap^{\mathfrak{F}} B = 0
\end{aligned}$$

(used the theorem 23). □