Consequently $\mathcal{A} \supseteq \operatorname{up}^{-1} R$.

Let now $\mathcal{B} \in \mathfrak{F}$ and $\forall \mathcal{A} \in S : \mathcal{A} \supseteq \mathcal{B}$. Then $\forall \mathcal{A} \in S : \operatorname{up} \mathcal{B} \supseteq \operatorname{up} \mathcal{A}$. $\operatorname{up} \mathcal{B} \supseteq \bigcup \langle \operatorname{up} \rangle S$. From this $\operatorname{up} \mathcal{B} \supseteq T$ for any finite set $T \subseteq \bigcup \langle \operatorname{up} \rangle S$. Consequently $\operatorname{up} \mathcal{B} \ni \bigcap^{\mathfrak{A}} T$. Thus $\operatorname{up} \mathcal{B} \supseteq R$; $\mathcal{B} \subseteq \operatorname{up}^{-1} R$.

Comparing we get $\bigcap^{\mathfrak{F}} S = \operatorname{up}^{-1} R$. □

Theorem 35 *If $\mathfrak{A}$ is a distributive lattice then for any $\mathcal{F}_0, \ldots, \mathcal{F}_m \in \mathfrak{F}$ ($m \in \mathbb{N}$)*

$$\operatorname{up}(\mathcal{F}_0 \cap^{\mathfrak{F}} \ldots \cap^{\mathfrak{F}} \mathcal{F}_m) = \left\{ K_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} K_m \mid K_i \in \operatorname{up} \mathcal{F}_i, i = 0, \ldots, m \right\}.$$

Proof Let's denote the right part of the equality to be proven as R. First we will prove that R is a filter. Obviously R is nonempty.

Let $A, B \in R$. Then $A = X_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} X_m$, $B = Y_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} Y_m$ where $X_i, Y_i \in \operatorname{up} \mathcal{F}_i$.

$$A \cap^{\mathfrak{A}} B = (X_0 \cap^{\mathfrak{A}} Y_0) \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} (X_m \cap^{\mathfrak{A}} Y_m),$$

consequently $A \cap^{\mathfrak{A}} B \in R$.

Let $R \ni C \supseteq A$.

$$C = A \cup^{\mathfrak{A}} C = (X_0 \cup^{\mathfrak{A}} C) \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} (X_m \cup^{\mathfrak{A}} C) \in R.$$

So R is a filter. Consequently the statement of our theorem is equivalent to

$$\mathcal{F}_0 \cap^{\mathfrak{F}} \ldots \cap^{\mathfrak{F}} \mathcal{F}_m = \operatorname{up}^{-1} R.$$

Let $P_i \in \operatorname{up} \mathcal{F}_i$. Then $P_i \in R$ because $P_i = (P_i \cup^{\mathfrak{A}} P_0) \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} (P_i \cup^{\mathfrak{A}} P_m)$. So $\operatorname{up} \mathcal{F}_i \subseteq R$; $\mathcal{F}_i \supseteq \operatorname{up}^{-1} R$.

Let now $\mathcal{B} \in \mathfrak{F}$ and $\forall i \in \{0, \ldots, m\} : \mathcal{F}_i \supseteq \mathcal{B}$. Then $\forall i \in \{0, \ldots, m\} : \operatorname{up} \mathcal{F}_i \subseteq \operatorname{up} \mathcal{B}$.

$L_i \in \operatorname{up} \mathcal{B}$ for any $L_i \in \operatorname{up} \mathcal{F}_i$. $L_0 \cap^{\mathfrak{A}} \ldots \cap^{\mathfrak{A}} L_m \in \operatorname{up} \mathcal{B}$. So $\operatorname{up} \mathcal{B} \supseteq R$; $\mathcal{B} \subseteq \operatorname{up}^{-1} R$.

So $\mathcal{F}_0 \cap^{\mathfrak{F}} \ldots \cap^{\mathfrak{F}} \mathcal{F}_m = \operatorname{up}^{-1} R$. □

Definition 57 *I will call a **lattice of filter objects on a set** a set of filter objects on the lattice of all subsets of a set. (From the above it follows that it is actually a complete lattice.)*

7.4. Distributivity of the lattice of filter objects

Theorem 36 *If $\mathfrak{A}$ is a distributive lattice with greatest element, $S \in \mathscr{P}\mathfrak{F}$ and $\mathcal{A} \in \mathfrak{F}$ then $\mathcal{A} \cup^{\mathfrak{F}} \bigcap^{\mathfrak{F}} S = \bigcap^{\mathfrak{F}} \left\langle \mathcal{A} \cup^{\mathfrak{F}} \right\rangle S$.*