A filter object $\mathcal{A}$ is represented by the value of $\operatorname{up}\mathcal{A}$. We are not interested in the internal structure of filter objects (which can be inferred from the appendix Appendix B), but only in the value of $\operatorname{up}\mathcal{A}$. Thus the name "filter objects" by analogy with an object in object oriented programming where an object is completely characterized by its methods, likewise a filter object $\mathcal{A}$ is completely characterized by $\operatorname{up}\mathcal{A}$.

7. Lattice of filter objects

7.1. Minimal and maximal f.o.

Obvious 18 *The filter object* $0 = \operatorname{up}^{-1}\mathfrak{A}$ *(equal to the least element of the poset* $\mathfrak{A}$ *if this least exists) is the least element of the poset of filter objects.*

Proposition 24 *If there exists greatest element* 1 *of the poset* $\mathfrak{A}$ *then it is also the greatest element of the poset of filter objects.*

Proof Take in account that filters are nonempty. □

Obvious 19 *1. If the base poset has least element, then the primary filtrator is down-aligned.*

2. If the base poset has greatest element, then the primary filtrator is up-aligned.

7.2. Primary filtrator is filtered

Theorem 31 *Every primary filtrator is filtered.*

Proof We need to prove that $\mathcal{A} = \bigcap^{\mathfrak{F}} \operatorname{up}\mathcal{A}$ for every $\mathcal{A} \in \mathfrak{F}$.

$\mathcal{A}$ is obviously a lower bound for $\operatorname{up}\mathcal{A}$.

Let $\mathcal{B}$ be a lower bound for $\operatorname{up}\mathcal{A}$ that is $\forall K \in \operatorname{up}\mathcal{A} : K \supseteq \mathcal{B}$. Then $\operatorname{up}\mathcal{A} \subseteq \operatorname{up}\mathcal{B}$; $\mathcal{A} \supseteq \mathcal{B}$. So $\mathcal{A}$ is the greatest lower bound of $\operatorname{up}\mathcal{A}$. □

7.3. Formulas for meets and joins of filter objects

Lemma 2 *If* f *is an order embedding from a poset* $\mathfrak{A}$ *to a complete lattice* $\mathfrak{B}$ *and* $S \in \mathcal{P}\mathfrak{A}$ *and exists such* $\mathcal{F} \in \mathfrak{A}$ *that* $f\mathcal{F} = \bigcup^{\mathfrak{B}} \langle f \rangle S$*, then* $\bigcup^{\mathfrak{A}} S$ *exists and* $f \bigcup^{\mathfrak{A}} S = \bigcup^{\mathfrak{B}} \langle f \rangle S$.

Proof f is an order isomorphism from $\mathfrak{A}$ to $\mathfrak{B}|_{\langle f \rangle \mathfrak{A}}$. $f\mathcal{F} \in \mathfrak{B}|_{\langle f \rangle \mathfrak{A}}$.

Consequently, $\bigcup^{\mathfrak{B}} \langle f \rangle S \in \mathfrak{B}|_{\langle f \rangle \mathfrak{A}}$ and $\bigcup^{\mathfrak{B}|_{\langle f \rangle \mathfrak{A}}} \langle f \rangle S = \bigcup^{\mathfrak{B}} \langle f \rangle S$.

$f \bigcup^{\mathfrak{A}} S = \bigcup^{\mathfrak{B}|_{\langle f \rangle \mathfrak{A}}} \langle f \rangle S$ because f is an order isomorphism.

Combining, $f \bigcup^{\mathfrak{A}} S = \bigcup^{\mathfrak{B}} \langle f \rangle S$. □

Theorem 32 *If* $\mathfrak{A}$ *is a meet-semilattice with greatest element* 1 *then* $\bigcup^{\mathfrak{F}} S$ *exists and* $\operatorname{up} \bigcup^{\mathfrak{F}} S = \bigcap^{\mathcal{P}\mathfrak{A}} \langle \operatorname{up} \rangle S$ *for every* $S \in \mathcal{P}\mathfrak{F}$.